Chapter 2

<u>Section 2.1: Solving Equations</u>

In this section we will use the "***Addition Principle***" and the ***"Multiplication Principle"*** to solve equations.

A. How to check if a solution really works for a given equation:

1. If y = 26 does x = 7?   3x + 5 = y?

2. If $y = \frac{8}{3}$, does $x = \frac{2}{3}$? $x + \frac{1}{3} = y$

3. **ALWAYS be sure to check your answers in the back of the book, or by plugging your solution into the original equation.**

B. The Addition and Multiplication Principles:

1. The ***Addition Principle*** states that you can _________________ the same number from/to both sides of an equation.
2. Record a couple of examples to demonstrate this fact:

3. The **Multiplication Principle** states that you can _____________ the same number on both sides of an equation.
4. Record a couple of examples to demonstrate this fact:

C. In solving equations, basically we are changing the original equation into successively simpler or equivalent equations that eventually lead us to an answer!

D. When solving equations, we are striving to isolate the variable on the left side of the equation with constants on the right side.

E. DO THESE PROBLEMS:

1. $y + 9 = 43$

2. $-6 = x - 11$

3. $t + \frac{3}{8} = \frac{5}{8}$

4. $y - \frac{3}{4} = \frac{5}{6}$

5. $\frac{y}{-8} = 11$

6. $\frac{-x}{6} = 9$

7. $-\frac{2}{5}x = -\frac{4}{15}$

8. $-0.2344x = 2028.732$

Section 2.2: Using the Principles Together

In this section we will use both the Addition and Multiplication Principles together in solving equations. We will also learn how to simplify an equation before using these two principles.

A. Some equations that involve both principles:

1. $3x + 6 = 30$
2. $-6z - 18 = -132$

B. Some equations that involve simplification before applying the principles: Note the summary of equation solving on page 94 of your text.

1. $6x + 19x = 100$
2. $8(2t + 1) = 4(7t + 7)$

C. Solving equations that involve Fractions: The beauty of solving equations is that you can "get rid of" any fractions in the equation. This then makes the solution process much easier! Work through the following examples to see how this process works.

1. $\frac{1}{2} + 4m = 3m - \frac{5}{2}$
2. $1 - \frac{2}{3}y = \frac{9}{5} - \frac{1}{5}y + \frac{3}{5}$

D. We can eliminate decimals from equations by following a similar process as we used to eliminate fractions. However, with the use of our calculators, we can usually solve decimal equations without the need to eliminate the decimals. (Note the next example)

1. $0.91 - 0.2z = 1.23 - 0.6z$

E. DO THESE PROBLEMS

1. $4x+3=-21$

2. $4+\frac{7}{2}x=-10$

3. $7+3x-6=3x+5-x$

4. $7(5x-2)=6(6x-1)$

5. $0.7(3x+6)=1.1-(x+2)$

6. A good way to solve this equation is to start by clearing fractions. How do you clear fractions?

$$\frac{1}{6}\left(\frac{3}{4}x-2\right)=-\frac{1}{5}$$

Section 2.3: Formulas

Many applications of mathematics are found in the use of Formulas. In this section we will explore how to use formulas to solve many types of problems.

A. Some sample problems that involve using formulas:

1. The Power rating "P", in Watts, of an electrical appliance is determined by the following formula: $P = I \cdot V$, where "I" is the current, in amperes, and "V" is the voltage, measured in volts. If a kitchen requires 30 amps of current and the voltage in the house is 115 volts, what is the wattage of the kitchen?

2. The surface area "A" of a cube with sides of length "s" is given by the following formula: $A = 6s^2$. Find the surface area of a cube with sides of 3 inches.

3. When all "n" teams in a league play every other team twice, a total of "N" games are played, where $N = n^2 - n$. If a soccer league has 7 teams and all teams play each other twice, how many games are played?

B. Many times it is helpful to solve a formula for a specific variable. The next few problems will give examples of how to do this.

1. Solve the following formula for "w": P = 2L + 2W

2. Solve the following for "c": $A = \frac{a+b+c}{3}$

3. Solve the following for "x": S = Ax + Bx

C. DO THESE PROBLEMS:

1. The area of a parallelogram is given by the formula: $A = b \bullet h$.
Assume you know both the area, A, and the height, h. What is formula for the base , b, in terms of area and height?

2. The velocity of sound is approximately 1000 feet/second in air. You are watching the Space Shuttle take off at Cape Canaveral. You see the ignition and eleven seconds later you hear the roar. How far away is the launch pad if the equation for the velocity of sound C, is distance , D, divided by time T. First solve the following equation for D.

$$C = \frac{D}{T}$$

3. The formula $A = P + Prt$ is used to compute the amount of money [A] in an account which is earning interest at rate [r] over a period of time [t]. Solve this formula for principle, [P].

Section 2.4: Applications with Percent

Working with Percents is something we all do as a natural part of our lives. In this section we will review how to use percents. See page 106 in your book.

A. Some important facts about percents:

1. PERCENT means parts per hundred. The phrase CENT refers to 100.

2. This little formula may be helpful $\frac{P}{100}=\frac{Sample}{Population}=\frac{IS}{OF}$

   Here P is Percent; Sample is a part of the Population. Note, the "Population" is merely a reference and the "Sample" may be larger or smaller than the population. Examine the problem. Sometimes $\frac{Sample}{Population}$ is the best; sometimes $\frac{IS}{OF}$ is more useful depending on how the problem is worded.

   EXAMPLE: In a certain group of 120 people, 15 are children. What Percent of this group are children?

   The Sample is 15 and the population is 120 so:

   ANS: $\frac{P}{100}=\frac{15}{120}$ So $P=\frac{15}{120}\bullet 100=12.5\%$

3. The fraction $\frac{P}{100}$ is the decimal equivalent of the Percent, P. So

$$Decimal\ Equiv=\frac{P}{100}$$

   Example: Change 25% to a decimal: $\frac{25}{100}=0.25$

4. And to convert a decimal to a Percent we rewrite the above formula:

$$P=Decimal\bullet 100$$

   Example: Convert .45 to a percent: $P=0.45\bullet 100=45\%$

B. **PROBLEMS:** Working sample problems that involve percents. Usually you can make a direct translation from the problem in English to Mathematics. Work through the next few problems to see how this is done.

1. What percent of 150 is 39?

2. 54 is 24% of what number?

3. What number is 1% of one million?

4. Convert to decimal notation: 125%

5. Convert to % notation: $\frac{4}{5}$

6. 7 is 175% of what number?

7. What is 2% of 40?

8. To obtain his degree, Frank must complete 125 hours (credits) of instruction. If he has already completed 60% of this requirement, how many more credits will he need?

Section 2.5: Problem Solving

One of the most important reasons for studying Algebra, is to use it in solving problems. This section will allow you to practice this very important skill. A mathematical word problem should not be read as though it is a story or as though it conveys some kind of useful information. A math word problem is a word image describing a relation amongst several elements. When ever possible use the statements in the problem to draw a picture then go to the problem to get information about the relevant elements of the picture.

A. The Five Steps to Problem Solving: Page 115
    1. Familiarize yourself with the problem: This may mean that you have to read the problem several times until you figure out what you need to do, and what information you have to do it! **ALWAYS: If possible, draw a picture!!** Underline the key words.
    2. Translate to Mathematical language. (This usually means that you define a variable to represent what you are trying to find and then write an equation that fits the situation)
    3. Carry out some mathematical manipulation. (In other words, solve the equation you created)
    4. Check your possible answer in the original problem. (Does you answer make sense in the given situation?)
    5. State the answer clearly. (Be sure to answer the given question. Don't just say that $x = 5$)

B. Look at the suggestions for becoming familiar with the problem on page 115 of your text. These are some very good suggestions as to how you might approach a problem solving situation.

C. **PROBLEMS:** Some sample problems for practice: (Be sure to read this section of your text very carefully and follow all of their examples as well)

    1. Twice the sum of 4 and some numbers is 34. What is the number?

    2. Doug paid $72 for a shockproof portable CD player during a 20% off sale. What was the original price?

3. The Iditarod sled-dog race extends for 1049 miles from Anchorage to Nome. If a musher is twice as far from Anchorage as from Nome, how many miles has the musher traveled?

4. The sum of two consecutive odd integers is 108. What are the integers?

5. In the world's oldest divorcing couple, the woman was 6 years younger than the man. Together, their ages totaled 188 years. How old were the man and the woman?

6. The second angle of a triangle is four times as large as the first. The third angle is 5 degrees more than the sum of the other two angles. Find the measure of the second angle. (Note: The sum of the angles of any triangle is always 180 degrees)

7. The top of the John Hancock Building in Chicago is a rectangle whose length is 60 feet more than the width. The perimeter is 520 feet. Find the width and the length of the rectangle. Find the area of the rectangle.

Section 2.6: Solving Inequalities

In this section we will learn how to solve an inequality. Basically you solve them just as you would an equation with one major exception that will be described below. See page 127 in your book.

A. Addition Property of Inequalities: You can add or subtract the same number from any inequality. Record some examples below to show why this is true.

1.

3.

2.

4.

B. Multiplication Property of Inequalities:

1. You can multiply or divide both sides of an inequality ***by any positive number***. Record some examples below.

2. IF you multiply or divide both sides of an inequality by a negative number, you must change the direction of the inequality. Record some examples below.

C. **PROBLEMS:** Graphing inequalities: Because inequalities usually have an infinite number of solutions, we often graph the solution sets. Practice this on the examples below. Also see page 128.

1: Graph $x > -2$

2: Graph $x \leq 5$

Solve and graph the solution sets for the following inequalities. Also write the solutions sets in set builder notation.

3: $y + 6 > 9$

4: $2x + 4 \leq x + 1$

5: $7 + 8x \geq 71$

6: $\dfrac{2}{3} - \dfrac{x}{5} < \dfrac{4}{15}$

7: $3(t-2) \geq 9(t+2)$

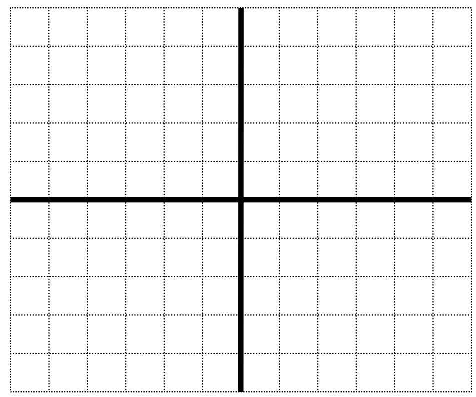
8: $\dfrac{4}{5}(3x+4) \leq 20$

Math 90 Lecture Notes
Chapter 3

<u>Section 3.1: Reading Graphs, Plotting Points, and Estimating Values</u>

In this section we will concentrate on graphing ordered pairs in the Rectangular Coordinate System, and take a quick look at a process known as Interpolation and Extrapolation.

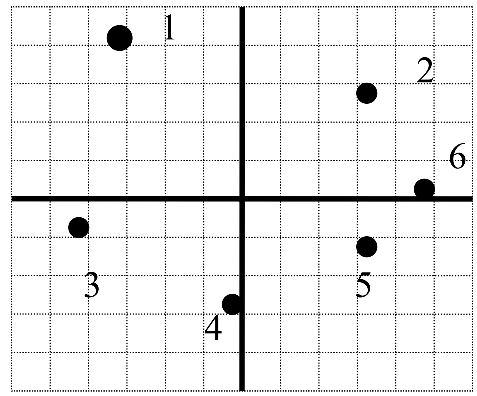
A. Graphing Ordered Pairs in the Rectangular Coordinate Plane:
   1. Label the following on the Coordinate Plane:
      a. The x-axis
      b. The y-axis
      c. The Origin
      d. The 4 Quadrants



   2. Points on the plane can be represented as "Ordered Pairs".
      a. The first number in an ordered pair is called the _____coordinate .
      b. The second number in an ordered pair is called the ____coordinate.
      c. An ordered pair is basically giving you directions from the Origin to another location in the plane. The x-coordinate tells you how far to go from the ____ axis, and the y-coordinate tells you how far to go from the _____axis .

   3. Graph the following points on the axis above: (2 , 5), (-2 , 1), (-2 , -5), (3 , -3)

4: List the Coordinates for the points graphed below:



1:______________

2:______________

3:______________

4:______________

5:______________

6:______________

B. DO THESE EXCERSIZES WITH ORDERED PAIRS

1: In which quadrant is each ordered pair located:

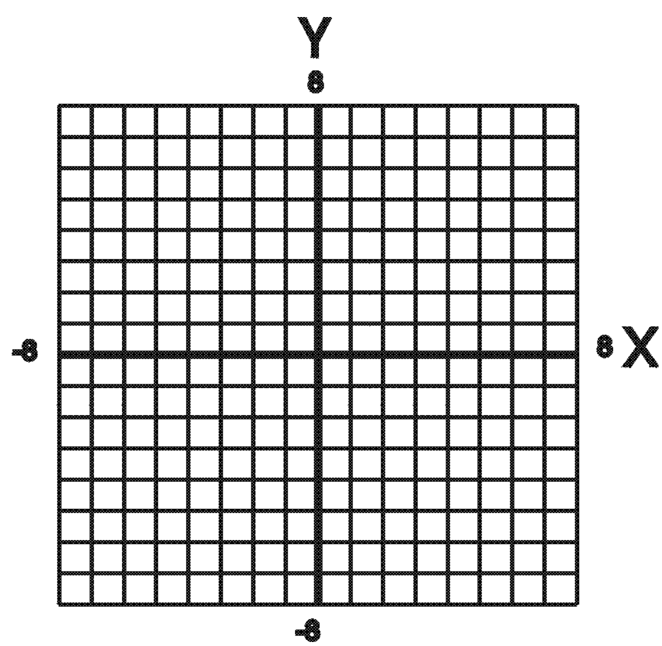
(7,-2)_____________

(-1,-4)____________

(-4,6)______________

(7.5,2.9)____________

2: Plot, and label, the following ordered pairs



A: (-7,-4)

B: (0,4)

C: (6,0)

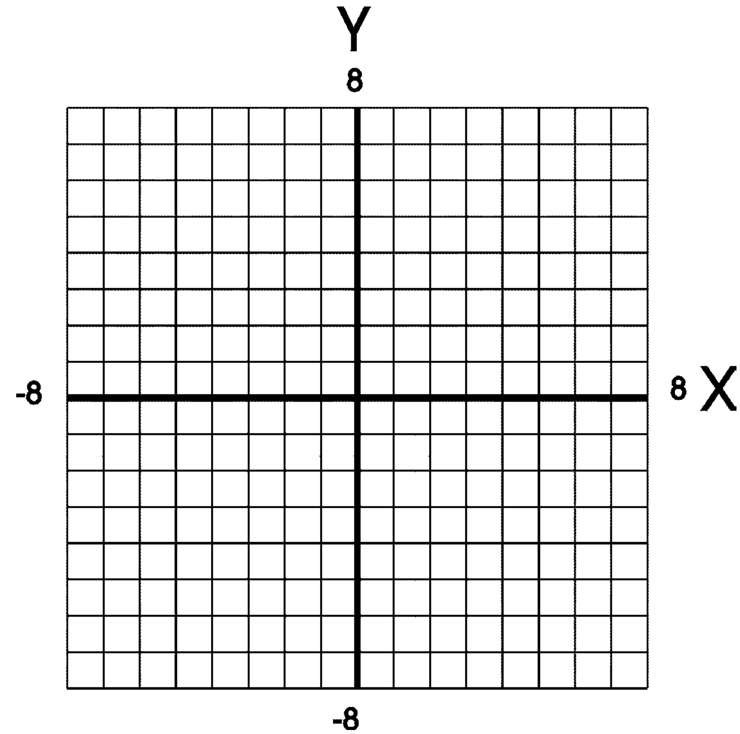
D: (5, -3)

3: The following ordered pairs are three corners of a rectangle. Plot these pairs and draw the rectangle.

(5,-2), (-3,-2),(-3,3)

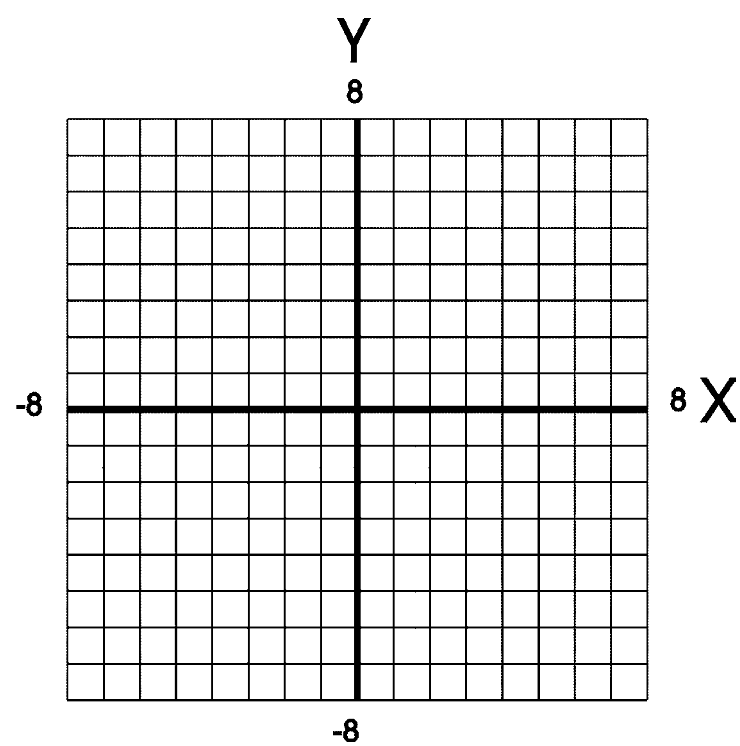
a) What is the missing ordered pair:

b) What is the perimeter of the rectangle.



4: Find the area of a triangle defined by the following ordered pairs.

(0,8), (0,-4), (5,-4)



Section 3.2: Graphing Linear Equations

A. Review: Finding and graphing solutions to equations in one variable. Solve and graph the solutions to the following:

1. $5x + 25 = 50$

2. $-3x + 21 = 33$

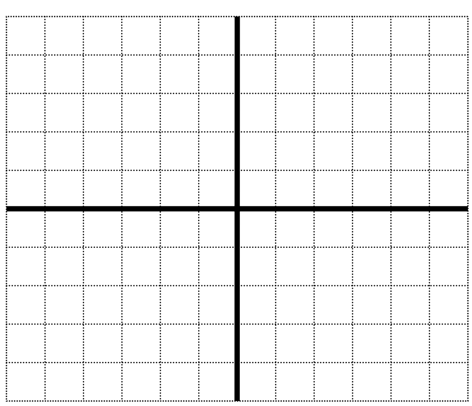
B. Finding solutions for equations with two variables:

1. Note that in both cases above, there was one variable and only one solution for each equation. This is often the case in equations with only one variable.
2. Find three possible solutions for the following equation: You must pick a value for one of the variables then solve for the other variable.

$$2x + y = 10$$

y	x	

3. Graph each point found above on the coordinate axis below:



What do you notice about the points you have graphed?

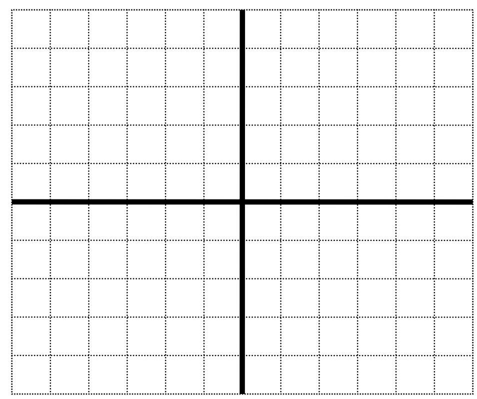
C. How to determine if a point is a solution to a given equation: Plug in the coordinates and then determine if you end up with a true statement. Work through the examples below to practice this skill.

1. Is the point (0 , 3) a solution to the following equation? $y = 2x + 3$

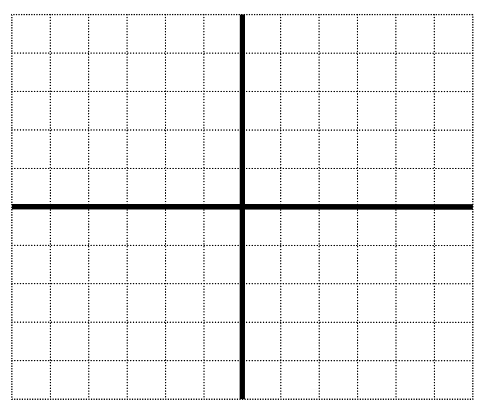
2. Is the point (0,5) a solution to the following equation? $5x - 3y = 15$

D. Graph the following equations by using the "Point-Plotting Method".

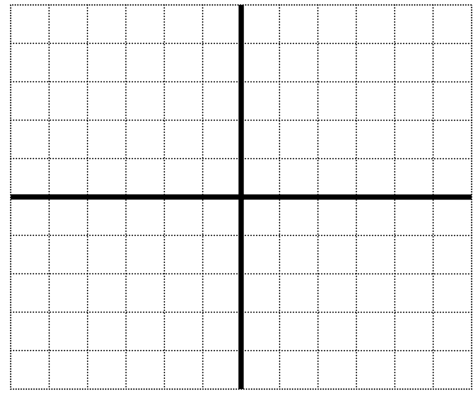
1. $Y = 2x + 2$



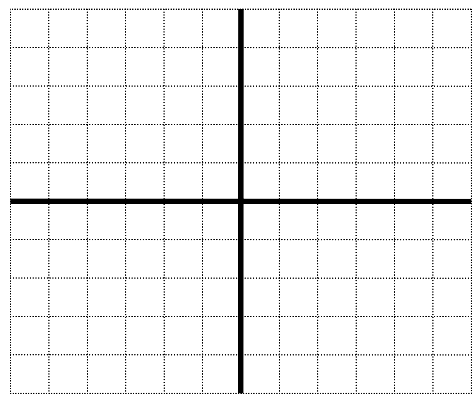
2. $y = \frac{1}{3}x$



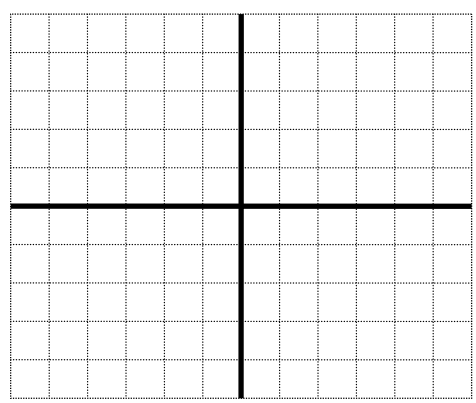
3. $y = \frac{5}{2}x + 3$



4. x + 2y = -6



5. 8x – 4y = 12

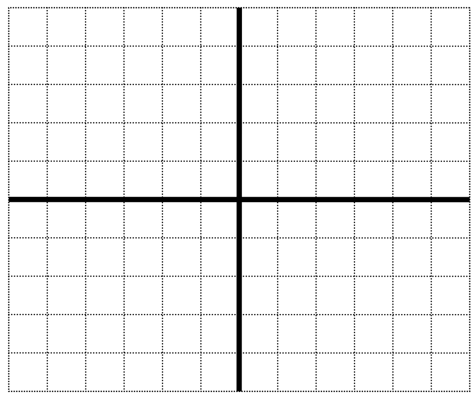


Section 3.3: Graphing and Intercepts

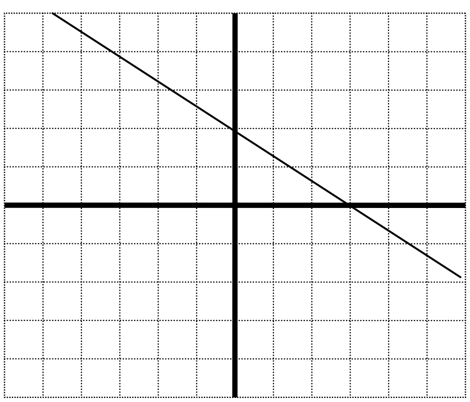
In this section we will learn how to graph linear equations using another method, actually an application of the Point-Plotting Method, called the Intercept Method. We will also learn how to graph horizontal and vertical lines.

A. Intercepts: The "x-intercept" is the point where a graph crosses the _____ axis. The "y-intercept" is the point where the graph crosses the _______ axis.

B. Sketch a graph below and point out the x and y-intercepts. Note that the x-coordinate of the y-intercept is always ________ . Also note that the y-coordinate of the x-intercept is always _______.

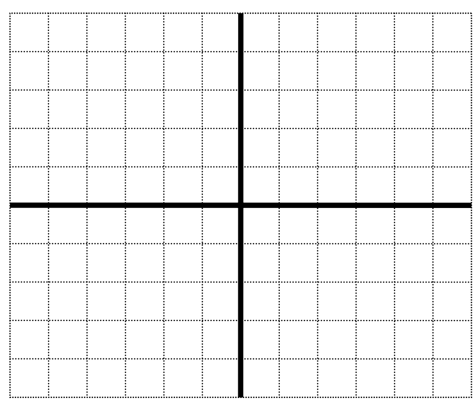


Find the x and y intercepts of the line in the graph.



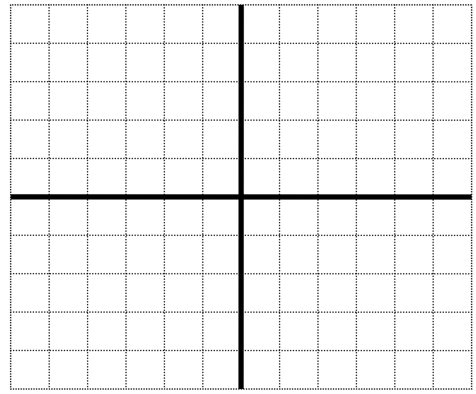
Plot this equation

$3x - 2y = 6$

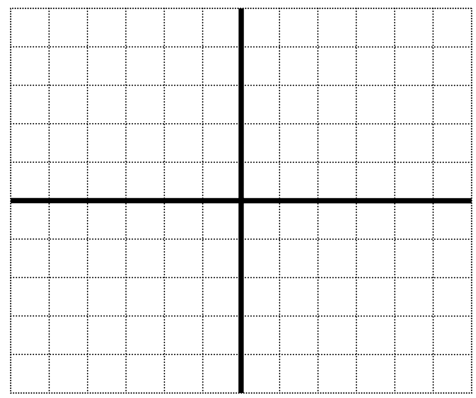


PLOT THESE EQUATIONS

3. $y = 2x - 6$

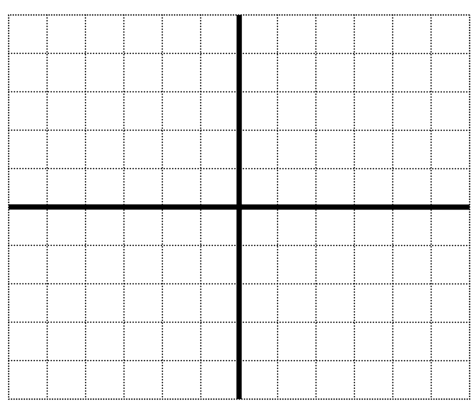


4. $3x - y = 2$

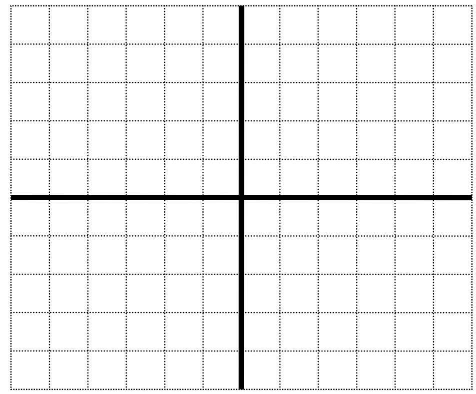


Graphing Vertical and Horizontal Lines: These lines are a little different than the ones you are used to graphing from previous problems.

5: Graph $y = 3$

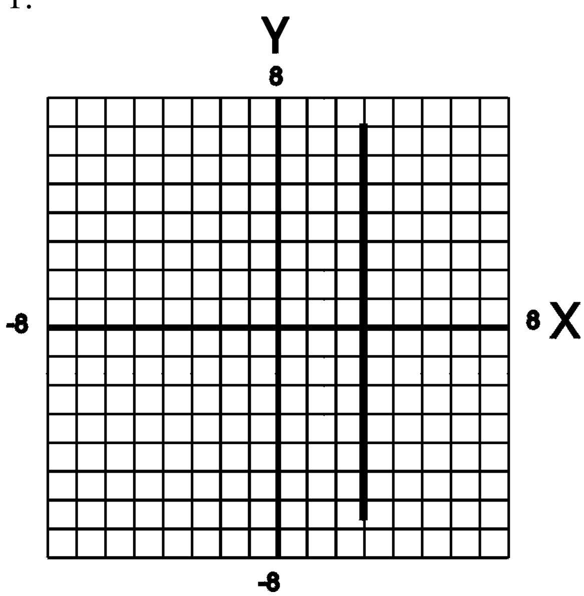


6: Graph x = -3

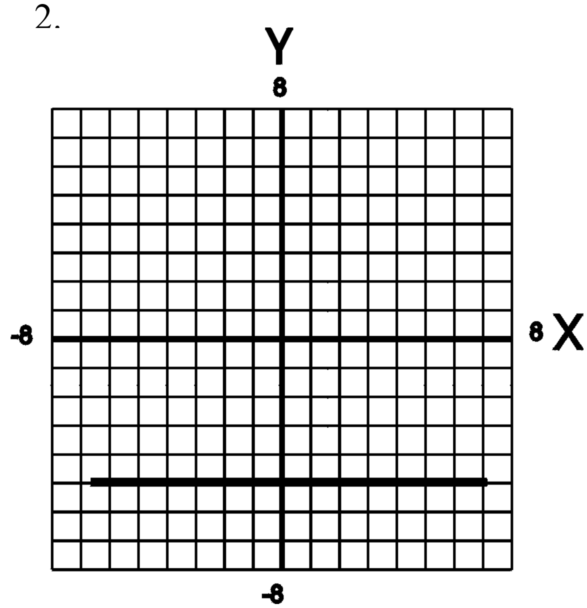


C: Write an equation for each of the following two graphs.

1.



2.



Chapter 3, Section 4: Rates of Change

In this section we will study how to measure rates of change. This is a very important application of slope as we continue our study of graphs.

A. Rates of Change: A ***"Rate"*** is a ratio that indicates how two quantities change with respect to each other! Discuss some familiar "rates" of change that you deal with in everyday life:

1. Speed in Miles per hour.

2.

3.

B. DO THESE PROBLEMS

1: On February 10, Oscar rented a Chevy Blazer with a full tank of gas and 13,091 miles on the odometer. On February 12, he returned the vehicle with 13,322 miles on the odometer. The rental agency charged $92 for the rental and needed 14 gallons of gas to fill the tank.

a) Find the Blazer's rate of gas consumption, in miles per gallon.

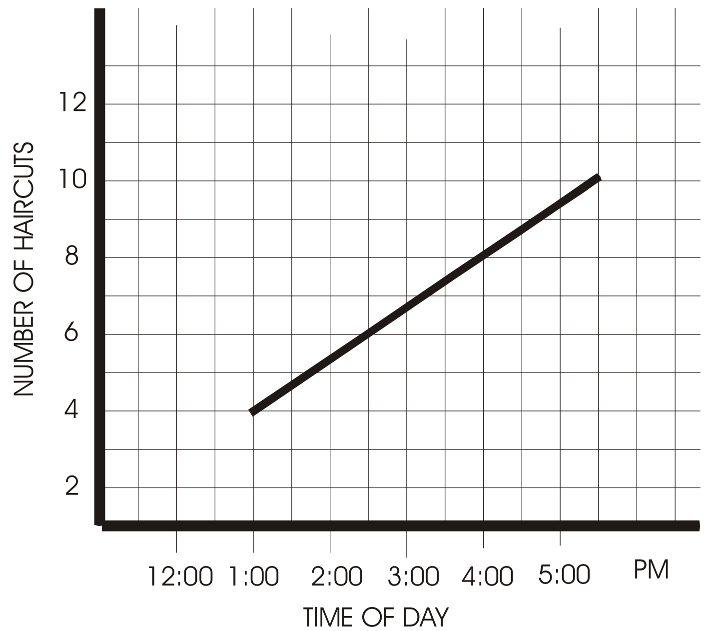
b) Find the average cost of the rental, in dollars per day.

c) Find the rate of travel, in miles per day.

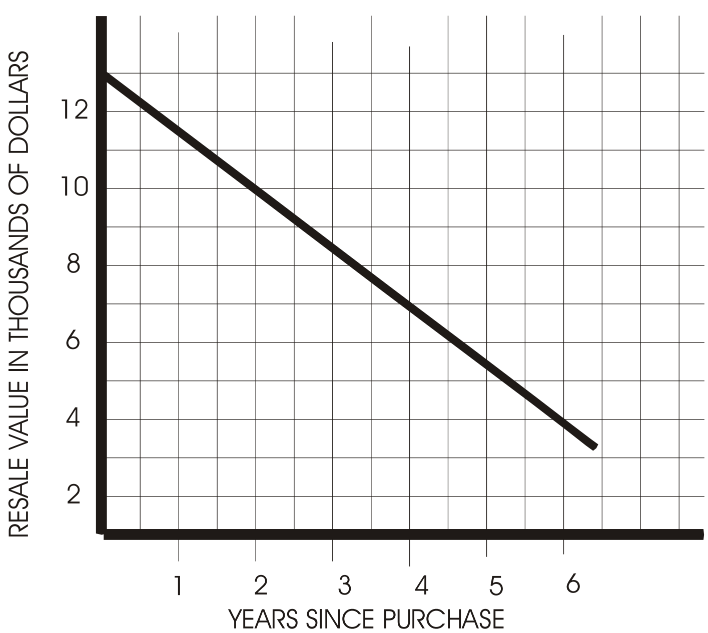
d) Find the rental rate, in cents per mile.

2: The tuition at a college was $1327 in 1999. In 2001 the tuition was $1359. What is the rate at which tuition is increasing?

3: Eve's Custom Cuts has a graph displaying data from a recent days work. At what rate does Eve work? Note that the hours scale on the horizontal axis caanot be used directly but must be converted to a time **difference.**



4: The graph below depicts the value of a car in thousands of dollars versus year since purchase. What is the rate of depreciation? Don't forget the algebraic sign.



Section 3.5: Slope

In this section we will continue our study of slope. Slope and "rate of change" are closely related. In fact, mathematically they are the same. The difference is that "rate of change" always has units (e.g. Miles/gallon) associated with it whereas slope may or may not have associated units.

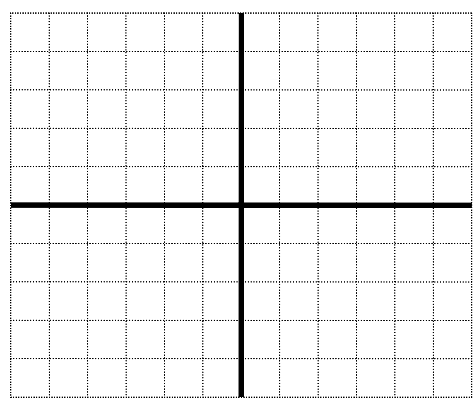
A. **VERY IMPORTANT:** Definition of slope:

1. Slope as a rate of change of the y-coordinate compared to the x-coordinate.
2. Slope = m = $\frac{Rise}{Run}$
3. Slope = m = $\frac{\Delta y}{\Delta x}$
4. Slope = m = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$

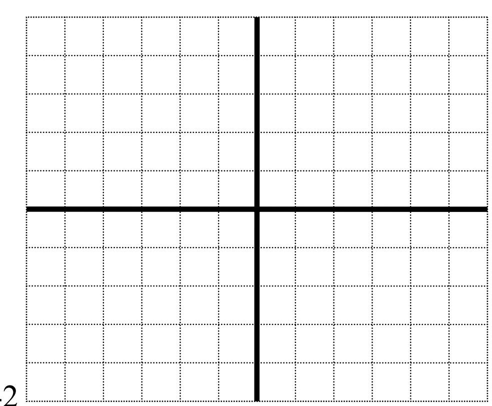
B. DO THESE PROBLEMS:

Find the slope of the line containing each given pair of points. If the slope is undefined, state this. Use the grids to check your answers.

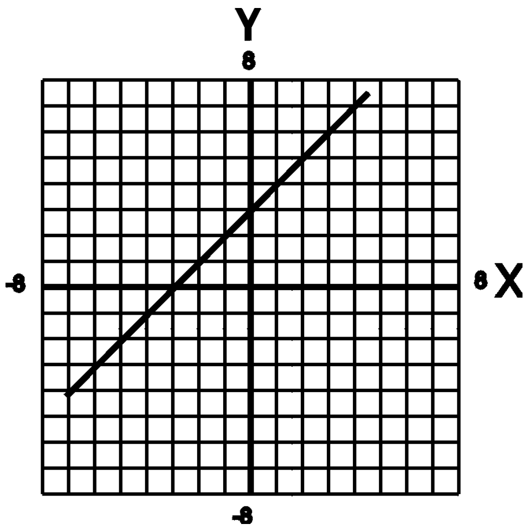
1. (2 , 1) and (–5 , 3)



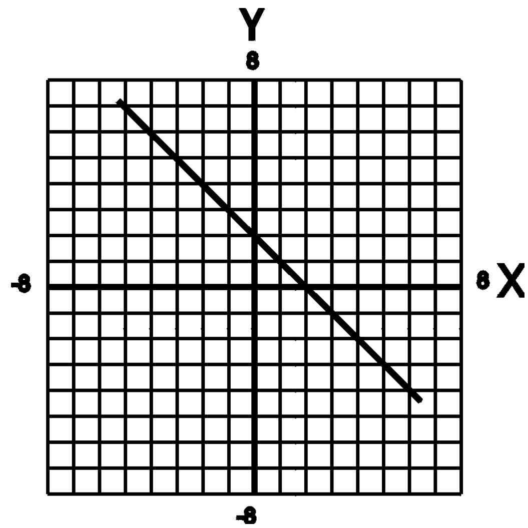
2. (–10 , 3) and (–10 , 4)



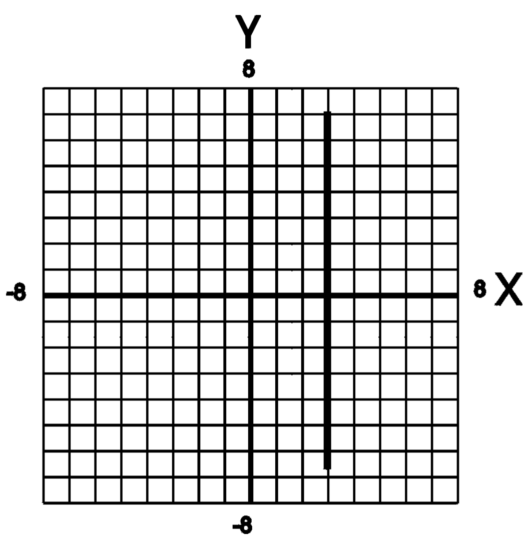
5: Write the slope of the line below each graph.



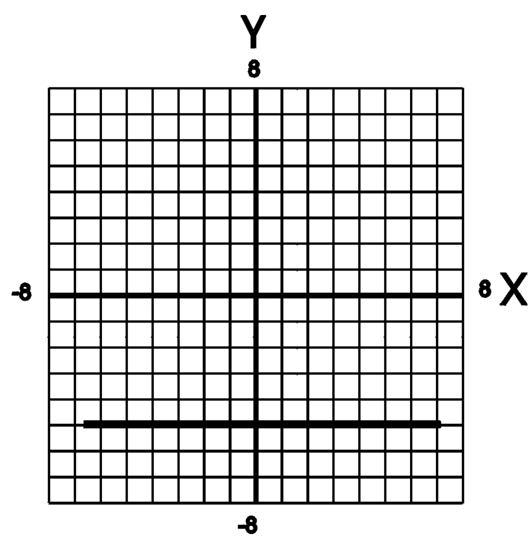
SLOPE:______________



SLOPE:______________



SLOPE:_____________

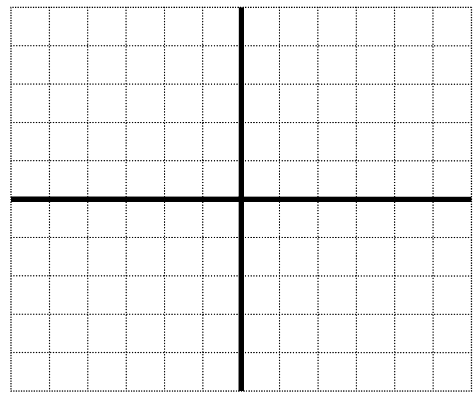


SLOPE:______________

Section 3.6: Slope-Intercept Form of the Equation

In this section we will learn how to use our knowledge of slope to simplify the graphing process.

A. Graph the following equation: 3y – 2x = -6



1. What is the slope of this line? _________

2. What is the y-intercept for this line? _______

3. Solve the above equation for y:

4. Note the y-intercept and slope in this equation!

B. The ***"Slope-Intercept"*** form of the equation:

1. Y = mx + b
2. "m" represents the ____________ of the line.

3. "b" represent the _______________________ of the line.

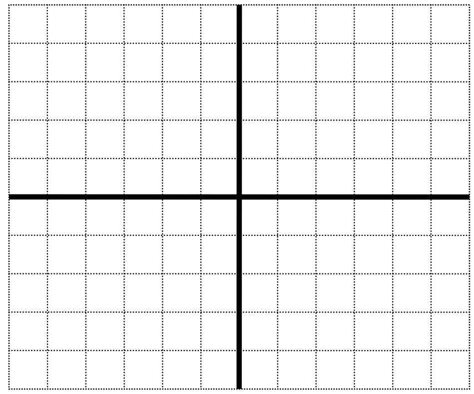
C. If the SLOPE is the same in a pair of equations then the lines plotted by those equations are PARALLEL. **Circle the constants in each equation telling us these two lines are parallel.**

$$y = -2x + 3; \qquad y = -2x - 4$$

D. DO THESE PROBLEMS

1. Draw a line that has the given slope and y-intercept:

   Slope = m = $\frac{2}{5}$, and y-intercept (0 , -3)



2. Find the slope and y-intercept for the following equation:

   $y = \frac{-3}{8}x + 6$

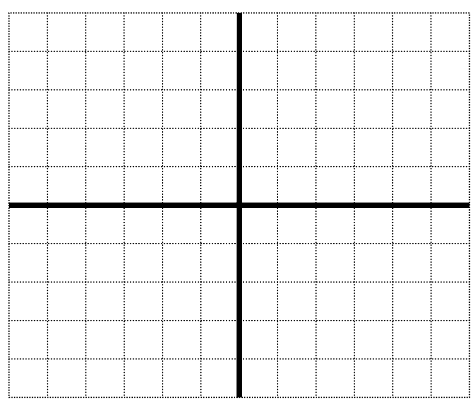
3. Find the slope and y-intercept for the following equation:

   $3x - 2y = 18$

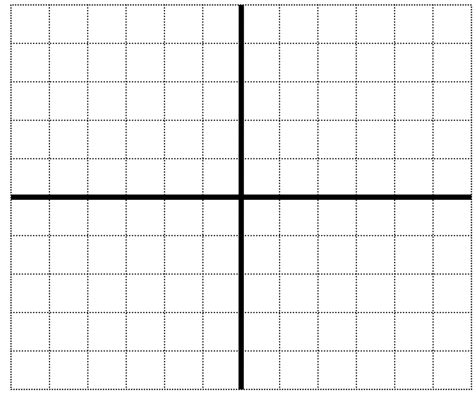
4. Graph the line with the following equation:

$$y = \frac{-3}{5}x - 1$$

$$y = \frac{-3}{5}x + 2$$



5. Graph the line with the given equation:
$4x + 5y = 15$ (First put equation in Slope-Intercept Form)



In the following problems, find the equation with the given slope and y-intercept:

6. Slope = m = -4, and y-intercept (0 , -2)

7. Slope = m = $\frac{3}{4}$, and y-intercept (0 , 23)

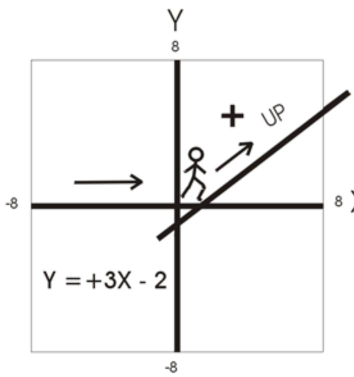
GETTING IT RIGHT

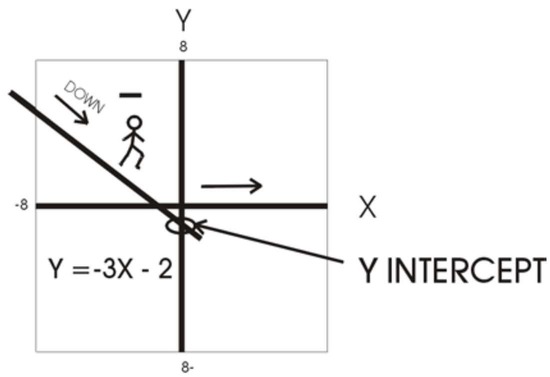
GRAPHS

1. REWRITE THE EQUATION IN SLOPE INTERCEPT FORM

$$Y = 3X - 2$$

SLOPE Y INTERCEPT





SLOPE: VISUALIZE A PERSON WALKING FROM LEFT TO RIGHT, JUST AS YOU READ, AND MEETING A HILL GOING UPHILL OR DOWNHILL.

UP-POSITIVE DOWN-NEGATIVE ← **MEMORIZE**

3. USING THE ORIGINAL EQUATION, PREPARE A TABLE OF X AND Y VALUES. THE TABLE SHOULD HAVE THREE POINTS.

FIRST POINT: LET X=0; CALCULATE Y
SECOND POINT: LET Y=0; CALCULATE X
THIRD POINT: PICK A NUMBER FOR X; CALCULATE Y

X	Y
0	-2
$\frac{2}{3}$	0
2	4

$Y = +3X - 2$

4. PLOT THE THREE POINTS. THEY SHOULD LIE ON A STRAIGHT LINE.

5. CHECK THE SLOPE AND Y INTERCEPT OF THE LINE AGAINST THE RESULTS YOU OBTAINED IN STEP 1.

Section 3.7: Point-Slope Form of a Linear Equation

In this section we will learn how to write equations of lines based on some information about that line. This is a very important skill, so learn it well.

A. Writing Equations of Lines: What do you need?

1. You need a ____________ that the line goes through.

2. You need to know the ___________ of the line.

3. If the above information is not known, you need to find it!

B.

1. When the unknown point is located between two known points, the process is known as ***"Interpolation".***

2. When the unknown point is located Outside two known points, the process is known as ***"Extrapolation".***

C. Work through the following problems to practice writing equations:

1. Write the equation of the line that goes through the following point, with the following slope: (6 , 2) , slope = m = 3

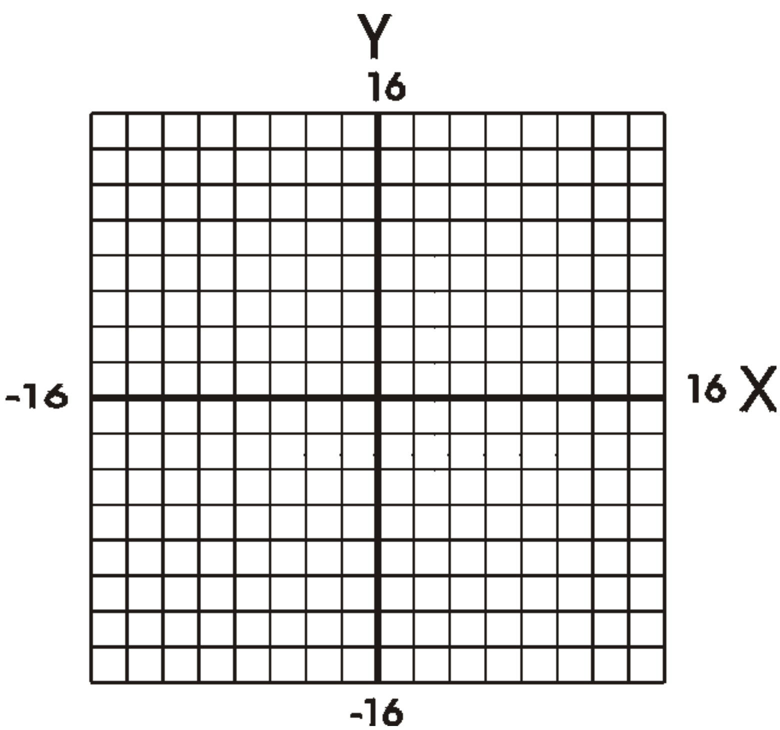
2. Write the equation of the line that goes through the following point, with the following slope: (4 , 7) , slope = m = $\frac{3}{2}$

3. Write the equation of the line that goes through the following two points: (3 , 7) and (4 , 8)

4. Write the equation of the line that goes through the following two points: (-2 , 3) and (2 , 5)

5: Write a point slope equation for the line with given slope and given point then plot the equation.

$m=\frac{5}{4};(-2,6)$



Math 90 Lecture Notes
Chapter 4 – Polynomials

Section 4.1: Exponents and their Properties

In this section we will learn how to work with exponents when applied to variable expressions. Learn this section very well as it lays the groundwork for the rest of the book.

A. Some background Principles (Do you remember this?)

1. x^5 : What do we call the "5" in this expression? ___________

   What do we call the "x" in this expression? ___________

   What does x^5 mean? _____________________________

B. Properties of Exponents: In this section of our notes, I am going to give some examples of how to work with expressions involving exponents. I want you to figure out the rules.

1. $x^3 \cdot x^4 =$

2. $y^2 \cdot y^3 \cdot y =$

3. $w^5 \cdot w^7 =$

4. What is the rule in this case? _______________________________

C. Find the rule for the following examples:

1. $\dfrac{x^5}{x^2} =$

2. $\dfrac{w^7}{w^3} =$

3. $\dfrac{y^3}{y} =$

4. What is the rule for handling the ratio of like variables raised to a power?

D. Figure this one out: (Use canceling and also use your rule for division)

1. $\dfrac{x^3}{x^3} =$

2. $\dfrac{y^5}{y^5} =$

3. $\dfrac{w}{w} =$

4. What is the rule?

E. How about this one?

1. $\left(x^2\right)^3 =$

2. $\left(w^5\right)^2 =$

3. $\left(y^4\right)^4 =$

4. What is the rule for this situation?

F. Figure this one out!

1. $(xy)^3 =$

2. $(2xy)^4 =$

3. $\left(3x^2w^3\right)^3 =$

What is the rule in this case?

G. Now do these problems.

1. $\left(\dfrac{x}{y}\right)^3 =$

2: $\left(\dfrac{2}{x^3}\right)^2 =$

3. $\left(\dfrac{2x^2y^3}{3w^2}\right)^4 =$

4: $n^3 \cdot n^{20} =$

5: $(2t)^8(2t)^{17} =$

6: $(a^8b^3)(a^4b) =$

7: $(a^3b)(ab)^4 =$

8: $\dfrac{x^{10}}{x^2} =$

9: $\dfrac{30n^7}{6n^3} =$

10: $\dfrac{a^{10}b^{12}}{a^2b^0} =$

11: $(-4)^0 - (-4)^1 =$

12: $(x^4y^6)(x^2y)^5 =$

13 $\left(\dfrac{x^3}{2y^2z}\right)^5 =$

Note: This is one section that you will have to give some time. Keep coming back to it and practice, practice, practice. Don't give up on it!

Section 4.2: Polynomials

In this section we will learn about ***Polynomials*** and how to work with them! We are beginning to bring together everything you have learned so far including factors, exponents, order of operations and fractions. Always put polynomials in ***standard form.*** That is the variable exponents are arranged in descending order.

A. Some important vocabulary:

1. ***Terms***: A term can be a number, a variable, or a product of numbers and/or variables which may be raised to powers.
2. Jot down some examples of terms below:

3. ***Polynomials***: A polynomial consists of one or more terms.

   a. The word "Poly" means: ______________________

   b. A monomial consists of ____________ term.

   c. A binomial consists of _____________ terms.

   d. A trinomial consists of _____________ terms.

   e. When a polynomial consists of more than 3 terms, we generally call it a _______________ .

4. The "***Degree***" of a term is the number of **variable** factors in that term. Also the sum of the exponents of all variable terms. Jot down some examples below. If there are no variable factors the degree is zero (0).

5. The Coefficient is a constant factor of the term: Jot down some examples below:

6. Look at Page 237 of your text to learn about the degree of a polynomial, leading term, and leading coefficient. Put the following polynomials in ***standard form:***

   a. $-3x^2+5x^5-2x+11+2x^3$

What is the degree of this polynomial? How many terms?

   b. $5y+y^5-6y^2+2y^4$

What is the degree of this polynomial? How many terms?

B. Combining "Like Terms": Just as you have learned in the past, you can only add, or combine terms that are alike, or similar. In the context of polynomials, ***Like or Similar terms are either constant terms or terms containing the same variable(s) raised to the same powers***. Simplify the following polynomials by combining like terms and write answer in descending order of the exponents.

1. $5a+7a^2+3a$
2. $8x^5-x^4+2x^5+5x^4-4x^4-x^6$
3. $5b^2-3b+7b^2-4b^3+4b-9b^2+10b^3$

C. Evaluate the following polynomials for the given value:

1. $4x^2-6x+9$ for x = 3
2. $5x-9+x^2$ for x = -2

D. DO THESE PROBLEMS

1. ***Skydiving application***: For jumps that exceed 13 seconds, the polynomial 173t – 169 can be used to approximate the distance, in feet, that a skydiver has fallen in "t" seconds. Approximately how far has a skydiver fallen 20 seconds after jumping from a plane?

2. ***Total Revenue***: Gigabytes Electronics is selling a new type of computer monitor. Total revenue is the total amount of money taken in. The firm estimates that for the monitor's first year, revenue from the sale of "x" monitors is: $250x - 0.5x^2$ dollars. What is the total revenue from the sale of 60 monitors?

3. Medical Dosage: The concentration, in parts per million, of a certain antibiotic in the bloodstream after "t" hours is given by the polynomial: $-.05t^2 + 2t + 2$ Find the concentration after 2 hours.

4. Rewrite the following polynomial in proper order then identify the terms, the coefficients of each term and the degree of each term.

$$7x^2 + 8x^5 - 4x^3 + 6 - \frac{1}{2}x^4$$

TERM	COEFFICIENT	DEGREE	DEG OF POLY

5. Evaluate each polynomial for x=3 and x=-3. Note the effect of odd exponents on odd numbers.

POLYNOMIAL	*Ans for* $x = 3$	*Ans for* $x = -3$
$-7x + 4$		
$2x^2 - 3x + 7$		
$\frac{1}{3}x^4 - 2x^3$		

Section 4.3: Addition and Subtraction of Polynomials

In this section we will learn how to add and subtract polynomials. When adding polynomials, we simply combine like terms and write answer in standard form.

A. Some examples of adding polynomials:

1. $(5x + 1) + (-9x + 4) =$

2. $\left(9a^2 + 4a - 5\right) + \left(6a^2 - 3a - 1\right) =$

3. $\left(7 + 4t - 5t^2 + 6t^3\right) + \left(2 + t + 6t^2 - 4t^3\right) =$

B. The opposite of a Polynomial: To find the opposite, or additive inverse, of a polynomial, simply change the sign of every term. This is the same as multiplying the polynomial by -1. Work through the following examples to practice this skill. This will help you learn how to subtract polynomials.

1. Write the opposite of $3x^2 - 2x + 4$ in two different forms:

2. Simplify the following: $-\left(-6x^3 + 2x^2 - 7\right)$

3. Simplify the following: $-\left(6x^5 - 2x^4 + 5x^3 - 6x^2 + 3x - 5\right)$

C. Work through the following problems to practice this skill.

1. $(5x + 6) - (-2x + 4) =$

2. $\left(-4x^2 + 2x\right) - \left(3x^3 - 5x^2 + 3\right) =$

3. $\left(7 + t - 5t^2 + 2t^3\right) - \left(1 + 2t - 4t^2 + 5t^3\right) =$

4. $\left(5x^3 - 4x^2 + 6\right) - \left(2x^3 + x^2 - x\right) + \left(x^3 - x\right) =$

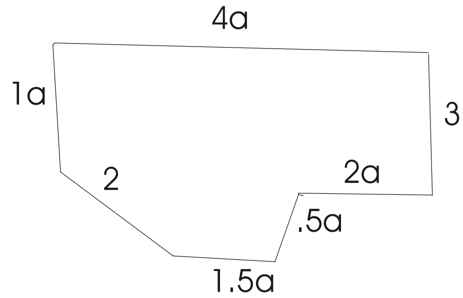
5. Write the additive opposite of the following expression with and without parenthesis.

$$-X^3 + 4X^2 - 9$$

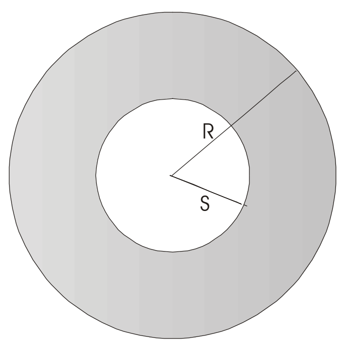
A:) WITH NO PARENTHESIS

B: WITH PARENTHESIS

6: Find the perimeter of the object below.



7: Find a polynomial for the shaded area.



Section 4.4: Multiplication of Polynomials

In this section we will learn how to multiply polynomials. To do this you will use the skills you have learned in previous sections.

A. Multiplying Monomials: Work through the following problems to practice this skill:

1. $\left(4x^3\right)7 =$

2. $\left(-x^6\right)\left(-x^2\right) =$

3. $\left(\frac{2}{3}x^2y^5\right)\left(\frac{-5}{3}xy^3\right) =$

4. $\left(-3ab^3c^5\right)\left(-5a^2b^4\right) =$

B. Multiplying a monomial and a polynomial together: To accomplish this, we simply ***apply the distributive property*** and remember all the rules of exponents we learned in this chapter.

1. Multiply: $5x(4x^3 - 5x^2 + x + 3) =$

2. Multiply: $3a^2b(a^3 + a^2b^2 + b^3) =$

C. When multiplying one polynomial by another, multiply each term in the first with each term in the second. When multiplying one binomial with another binomial we can use what is commonly referred to as the FOIL method of multiplication: FOIL stands for the following:

F: ____________________
O: ____________________
I: _____________________
L: ______________________

1. Try the FOIL method on the following products:
   a. $(2x + 3)(x - 5)$

   b. $(x - 2)(x + 7)$

   c. $(5x + 1)(2x + 3)$

D. The book shows you a method of multiplication that they call the Column Method. I will show you a different method that I call the Chart or ***Table method***. Using the Table/Chart method works very well when we are multiplying polynomials that are larger than binomials, but will work for the product of binomials as well.

Use the Table method to multiply the following:

1: $(2x - 3)(3x + 5)$

2: $(2x + 3)(x^2 - 6x + 5)$

3: $(x^2 - 3x + 7)(3x^2 + 4x - 2)$

4: MULTIPLY AND CHECK: $\left(\frac{1}{4}x + 2\right)\left(\frac{3}{4}x - 1\right)$

5: MULTIPLY AND CHECK: $(3x + 2)(5x + 7 + 4x)$

Section 4.5: Special Products

In the last section we learned how to multiply polynomials together using the FOIL method and using a "Table" method. In this section we will learn some short cuts for multiplying special types of polynomials. (There are some tricks we can use to speed up the multiplication process). **However, you do not have to use the tricks. You can always go back and use the FOIL process, or the "Table" method to do your multiplication!!!** Note there are many kinds of special products in mathematics. We are discussing only a few in this course.

A. Finding the square of a binomial – some examples: Square the following binomials using either the "Table" method or the FOIL method.

1. $(x + 5)^2$

2. $(2x - 3)^2$

3. $(5x + 2)^2$

4. Do you notice a "pattern" that you could use to square a binomial in your head, without having to write everything out? What is the pattern?

5. A common mistake: Prove that $(x+y)^2 \neq x^2 + y^2$

B. Another special product that you see quite often is the following called a ‘difference of squares”. Multiply the following using either the FOIL method or the “Table”. Note this ‘difference of squares’ product is a favorite of algebra teachers. It springs up frequently in algebra tests in many forms and recognizing it can save you an enormous amount of time and effort.

1. $(x - y)(x + y)$

2. $(a - 6)(a + 6)$

3. $(5x + 2)(5x - 2)$

Do you notice a pattern in these products? What is the pattern?

C. Do these problems

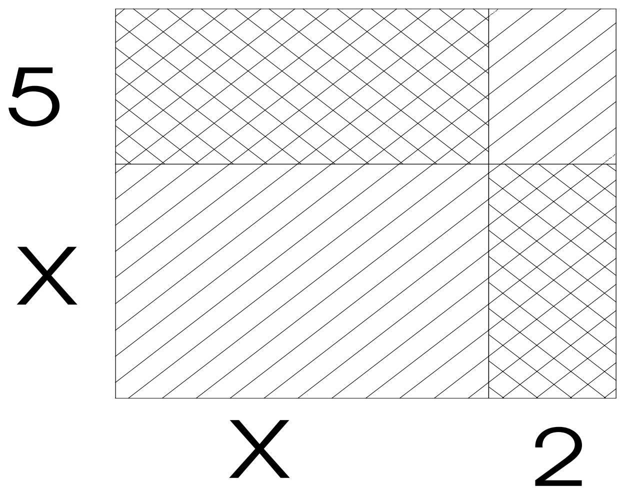
1. Multiply $(a+9)(a-9)$

2. Many math teachers love the following problem. Look carefully and you will see the special products we just discussed.

$$(4X^2+9)(2X+3)(2X-3)$$

3. Multiply $\left(7X^3+1\right)^2$

4. Compute total area of the figure by two methods. One method is simply height times width. The other method is to compute each sub-area individually and add.



Section 4.6: Polynomials in Several Variables

In this section, we will learn how to work with polynomials that have several variables.

A. Evaluation Polynomials: Work through the following examples to learn how to evaluate polynomials that have more than one variable.

1. Evaluate: $x^2 - 3y^2 + 2xy$, for x = 5, and y = -2

2. The altitude of an object, in meters, is given by the following polynomial: $h + vt - 4.9t^2$. A golf ball is thrown upward with an initial speed of 30 meters per second by a golfer atop the Washington Monument, 160 meters above the ground. How high above the ground will the ball be after 3 seconds?

B. Like Terms and Degree: Terms that are alike have the same variable and exponent parts. The only part of the term that can be different is the coefficient. Work through the following problems to practice this skill.

1. Identify the coefficient and degree of each term of the following polynomial: $-5x^3y^2 + 3x^5y^3 - 11xy^4$

2. Combine the “like” terms for the following.

   a) $3xy + 4x^2y^2 - 7xy + 3x^2y^2$

   b) $-2w^2h^3 - 5wh^4 + 8h^3w^2$

We are working to get comfortable working with expressions containing a mix of variables and constants.

C. DO THESE PROBLEMS

1: Evaluate for x=2;y=3 $\left(2x^2y+1\right)^2$

2: Identify the coefficient and degree of each term then find the degree of the polynomial

$$x^3y-2xy+3x^2-5$$

Term	Degree	coefficient	Deg of polynomial
x^3y			
$-2xy$			
$3x^2$			
-5			

Addition and Subtraction of polynomials: Work through the following problems to practice this skill with polynomials in several variables:

3: $\left(2r^3+3rs-5s^2\right)-\left(5r^3+rs+4s^2\right)$

4: $\left(2x^2-3xy+y^2\right)+\left(-4x^2-6xy-y^2\right)$

5: $\left(a^3+b^3\right)-\left(-5a^3+2a^2b-ab^2+3b^3\right)$

Multiplication of these types of polynomials is very similar to the multiplication you learned in the last couple of sections. You can use the FOIL method, or use a table to help you. Work through the following problems to practice this important skill. **LOOK FOR THE SPECIAL PRODUCTS IN SOME OF THESE PROBLEMS!**

6: $(5x+y)(2x-3y)$

7: $(a-3b)(a+3b)$

8: $\left(c-c^2d^2\right)\left(4+c^2d^2\right)$

9: $(a+b+c)(a-b-c)$

Section 4.7: Division of Polynomials

In this section we will learn how to divide polynomials by monomials, and also by binomials. Here we are trying to develop our skills recognizing what can be divided and what cannot be divided.

A. Division of a polynomial by a monomial: Work through the following examples to see how this is done.

1. $\left(12a^4 - 3a^2\right) \div 6$

2. $\dfrac{20t^3 - 15t^2 + 30t}{5t}$

3. $\dfrac{18t^6 - 27t^5 - 3t^3}{9t^3}$

B. Dividing a polynomial by a binomial. Before launching into long division you should first try to factor the numerator. If the numerator caanot be factored then do long division.

1. $\left(x^2-6x+8\right)\div\left(x-4\right)$

2. $\dfrac{t^2+8t-15}{t+4}$

3. $\dfrac{10x^2+13x-3}{5x-1}$

4: $\left(18x^3-24x^2+6x\right)\div(3x)$

5: $$\frac{\left(4x^{7}+6x^{5}+4x^{4}\right)}{\left(4x^{3}\right)}$$

6: This one has a remainder $\left(x^{4}-2x^{2}+4x-5\right)\div\left(x-1\right)$

7: $\left(x^{2}-x-20\right)\div\left(x-5\right)$

8. $\left(45x^{8k}+30x^{6k}-60x^{4k}\right)\div\left(3x^{2k}\right)$

Section 4.8: Negative Exponents and Scientific Notation

In this section we will expand our knowledge of exponents to include the use of negative exponents. Remember that negative exponents still reflect the rules of exponents that we have already studied. We will also review the use of Scientific Notation.

The table to the right shows what happens as a positive exponent decreases, passes through zero and then goes negative. Of particular importance is the fact that ANY number (except 0) or any variable raised to the zero power is 1. Furthermore the value of a base raised to a specific negative power is the reciprocal of that base raised to the same specific positive power.

2^3	8
2^2	4
2^1	2
2^0	1
2^{-1}	½
2^{-2}	¼
2^{-3}	1/8

NOTE THAT NEGATIVE EXPONENTS RESULT IN RECIPROCALS

A. Work through the following to learn about negative exponents:

1. Find $x^0 =$
2. Give an (division) example to show why the above is true:
3. Record the exponent rule for division:
4. Use this rule to simplify the following: $\dfrac{x^4}{x^7} =$
5. Complete: $x^{-n} =$
6. Complete the following table to show why this new rule works:

5^3	125
5^2	25
5^1	5

B. Note the following rules in your text that are natural consequences of the rule for negative exponents:

1. See page 286, Yellow Box of your text: $\frac{a^{-n}}{b^{-m}} =$

2. See page 287, Yellow Box in your text: $\left(\frac{a}{b}\right)^{-n} =$

3. Note the summary of exponent rules in your text on page 288.

C. DO THESE PROBLEMS

1: EXPRESS USING POSITIVE EXPONENTS

a: 7^{-2}

b: $\left(\frac{3}{5}\right)^{-2}$

2: EXPRESS USING NEGATIVE EXPONENTS

a: $\frac{1}{6^2}$

b: $\frac{1}{m}$

3: SIMPLIFY. DO NOT USE NEGATIVE EXPONENTS IN THE ANSWER.

a) $\left(a^{-2}\right)^9$

b: $\frac{y^{-7}}{y^{-3}}$

c: $\frac{3y^4}{s^{-2}y^{-4}}$

D. Scientific Notation: Scientific Notation is used when we are working with very small numbers, or very large numbers. See page 288 for the definition of Scientific Notation. With Scientific Notation all numbers are expressed in the form:

$$N \times 10^m$$
$$\textit{where } 1.0 \leq N < 10$$

Work through the following problems to practice writing numbers in Scientific Notation, and converting numbers from Scientific Notation into standard notation.

1. Convert the following into Scientific Notation:
   a) 27,000

   b) .00568

   c) 360

   d) .000212

2. Convert the following numbers back to standard notation:
   a) $3.77x10^7$

   b) $2.25x10^{-4}$

   c) $1.2x10^{-7}$

   d) $2.365x10^6$

E. Work through the following problems to learn how to multiply and divide using Scientific Notation:

1. $\left(1.9x10^{8}\right)\left(3.4x10^{-3}\right)$

2. $\left(4.7x10^{5}\right)\left(6.2x10^{-12}\right)$

3. $\left(5.6x10^{-2}\right)\left(2.5x10^{5}\right)$

4. $\left(7.5x10^{-9}\right)\left(2.5x10^{12}\right)$

Section 5.1: Introduction to Factoring

This is the chapter in which we learn the very important skill of factoring polynomials, especially trinomials. Please remember that when we ask you to factor something, we are asking you to ***write it as a product***. We start out by learning how to factor out the greatest common factor from an expression.

A. A Little Review: Multiply the following:

1. $7(2x + 5)$
2. $2x(5x^2 + 3)$
3. $3x^2(5x - 7)$
4. $3x^3(2x^2 - 3x + 5)$

B. Factoring out the greatest common factor is just the opposite of what you did in problems #1 – 4 above. We start with the answer and work backwards, writing it as a product. Factor out the greatest common factor for the following problems.

1. $14x + 21$
2. $7x^3 - 4x^2$
3. $16x^4 - 20x^2 - 16x$
4. $5ab^2 + 10a^2b^2 + 15a^2b$

5. $49xy - 21x^2y^2 + 35x^3y^3$

C. **Factoring by Grouping**: ***We will use this technique when we have 4 terms that we want to factor***. Sometimes polynomials with four terms contain binomial factors. In these cases, we group the first two terms and factor out the greatest common factor from them, then we group the second two terms and factor out the greatest common factor from them. Hopefully at this point, we have a common factor that will allow us to factor the original expression. Work through the following examples.

These types of problems are best manipulated mentally before writing anything down. Look at this first problem- here we see a common x in the first two terms. If we factor out the x then we are left with y+2. Now look at the second pair of terms. Here we see a common 4 and, if we factor that out, we are left with y+2 for a factor of the second two terms. So x+2 is the common binomial factor of the this four term polynomial.

1. $xy + 2x + 4y + 8$

2. $3ax + 21x - a - 7$

3. $x^3 - 5x^2 - 4x + 20$

4. $8x^3 - 12x^2 + 14x - 21$

Section 5.2: Factoring Trinomials of the type $x^2 + bx + c$

In this section of the text, we learn how to factor trinomials. This is one of the most important parts of this course! The ability to factor will make or break you in the next algebra course that you take. The text shows you a method of trial and error for factoring.

See pages 80 and 84 of this book for guidance on determining the algebraic signs in the factors.

Successful factoring requires PRACTICE, PRACTICE, PRACTICE!!!

Plan to spend some time on this concept – it will pay off "***BIG TIME***" later in this course and in the future!

A. A review of multiplication using the table: Multiply the following using the table:

1. $(x + 7)(x - 5)$

2. $(x + 2)(x + 6)$

B. Factor the following:

FACTORS OF 12

CHECK BY MULTIPLYING

1. $x^2 + x - 12$

i. Determine the signs
ii. List the factors of 12
iii. Enter the two selected factors
iv. Check ans.

$$x = (x \qquad)(\,x \qquad)$$

2. $x^2 - 5x - 24$

FACTORS OF 24

CHECK BY MULTIPLYING

$$x = (x \qquad)(x \qquad)$$

3. $x^2 + 8x + 12$

FACTORS OF 12

CHECK BY MULTIPLYING

$$x = (x \qquad)(x \qquad)$$

4. $x^2 + x - 42$

FACTORS OF 42

CHECK BY MULTIPLYING

$$x = (x \qquad)(x \qquad)$$

5: Factor Completely: $x^4 - 11x^3 + 24x^2$ (Make your own table)

6: Factor Completely: $3x^3 - 3x^2 - 18x$ (Make your own table)

7: $6x - 72 + x^2$ (First put in standard form, make own table)

8: $11 - 3w + w^2$ (A polynomial that will not factor is called "***Prime***")

GET IT RIGHT
FACTORING

$$X^2 + BX + C$$

1. **REARRAGE TERMS IN PROPER ORDER:** $X^2 + X - 42$

2. **PLACE THE PARENTHESIS:** $X^2 + X - 42 = (\quad\quad)(\quad\quad)$

3. **PUT THE X'S IN PLACE:** $X^2 + X - 42 = (\ X \quad\quad)(\ X \quad\quad)$

4. **NOW THE AGEBRAIC SIGNS.** THE SIGNS CAN BE FOUND BY EXAMINING THE QUADRATIC. **THERE ARE ONLY FOUR CASES.**

$X^2 + BX + C = (X + a)(x + b)$ P○P = P○P	$X^2 + BX - C = (X + a)(x - b)$ P LU M = P LU M
$X^2 - BX + C = (X - a)(x - b)$ M○P = M○M	$X^2 - BX - C = (X - a)(x + b)$ M○M = M○P

In the above figure the P's stand for Plus and the M for Minus. Imagine PoP (dad) sitting on a couch with a soda PoP in one hand, a bowl of PluMs in the other and MoM standing nearby with a MoP ready to clean up the mess.

In this case the signs are Plus and Minus: $X^2 + X - 42 = (\ X + \quad\quad)(\ X - \quad\quad)$

5. **Now the numbers. The product of the two numbers must equal the constant in the quadratic and the sum must be the coefficient of middle term . So- on a piece of paper, list the prime factors of the constant and include 1 as a factor. For 42 these factors are 1,2,3,7. From these values make all possible pairs as shown at the right.**

$1,42$
$2,21$
$3,14$
$6,7$

6. **The correct factors are 6 and 7 since the product is 42 and the difference is 1.**

$$X^2 + X - 42 = (\ X + \ 7\ )(\ X - \ 6\ )$$

7. **The final , most important step, is to check your answer by multiplication!**

Section 5.3: Factoring Trinomials of the Type $ax^2 + bx + c$

In this section we practice how to factor trinomials that have a leading coefficient other than one. These usually take a little more time and effort to figure out. We will also work on factoring trinomials that require us to factor out the greatest common factor first, before we factor the trinomial. Learn these skills well!!!

There are two ways to approach problems like these:

a) Trial and error: Try all combinations of the factors of the first and last terms. Can be very time consuming but if you stick with it you will eventually arrive at the correct answer.

b) Factor by grouping approach: This is much faster but requires that you remember the various steps.

A. Some practice problems:

1. $2x^2 + 5x + 3$

2. $3x^2 - 2x - 5$

3. $6x^2 - 13x + 6$

4. $4x^2 + 12xy + 9y^2$ (Make your own table for this one)

5. $4x^2 - 4x - 15$ (Make your own table for this one)

B. Factoring problems where you need to factor out the greatest common factor first – some examples:

1. $6x^2 - 51x + 63$

2. $18a^2 + 48a + 32$

3. $10x^4 + 7x^3 - 12x^2$

4. $6x^3 + 19x^2 + 10x$

GETTING IT RIGHT
SIGNS IN THE FACTORS

$$x^2 \quad -2x \quad -15$$

PRODUCT

$$(x \quad +3)(x \quad -5)$$

$$x^2 \quad -2x \quad -15$$

SUM

$$(x \quad +3)(x \quad -5)$$

Look at the sign of the last term in the quadratic. If it is negative then one of the factors must be POSITIVE; the other NEGATIVE. This means the sign of the middle term can be either PLUS or MINUS depending on the size of the constants in the factors.

$$x^2 \quad -8x \quad +15$$

PRODUCT

$$(x \quad -3)(x \quad -5)$$

$$x^2 \quad -8x \quad +15$$

SUM

$$(x \quad -3)(x \quad -5)$$

If the sign of the last term in the quadratic is PLUS then the signs in the factors must both be plus **OR** both must be negative. If the sign of the middle term in the quadratic is plus then both factors are plus (bottom figure); if the sign of the middle term in the quadratic is minus then both factors are minus (top figure).

$$x^2 \quad +8x \quad +15$$

PRODUCT

$$(x \quad +3)(x \quad +5)$$

$$x^2 \quad +8x \quad +15$$

SUM

$$(x \quad +3)(x \quad +5)$$

Section 5.4: Factoring Perfect-Square Trinomials and Diff. of Squares

In this section we introduce you to a special case in factoring known as the difference between two squares. We have already looked at this subject back in Section 4.5. These are very easy to factor once you recognize them. We will also look at factoring perfect-square trinomials.

A. A quick review of multiplication - Multiply the following:
   1. $(x - 5)(x + 5) =$

   2. $(2x - 7)(2x + 7) =$

   3. $(3x + 2)(3x - 2) =$

   4. Do you remember the pattern in the 3 products given above?

B. Using the pattern you see above, how would you factor the following? You won't need the table to factor these special cases, but you could use it if you wanted to do so.
   1. $x^2 - 49 =$

   2. $y^2 - 64 =$

   3. $9x^2 - 36 =$

4. $81x^2 - 1 =$

5. $4w^2 - 25 =$

C. Factoring Perfect Squares: Use the table to factor the following:

1. $x^2 + 6x + 9 =$

2. $m^2 + 12m + 36 =$

3. $25x^2 + 10xy + y^2 =$ (Make your own table here)

$$9x^2 - 49$$

NEG

PERF SQUARE TERM

PERF SQUARE TERM

TNT

TNT BLOWS UP-IT CAN BE FACTORED
AND EQUALS (3X + 7)(3X - 7)

$$9x^2 + 49$$

POS

PERF SQUARE TERM

PERF SQUARE TERM

TPT

TPT DOES NOTHING- P MEANS PHIZZLE
THERE ARE NO FACTORS !

Section 5.5: Factoring: A General Strategy

In this section of the text we simply give you a lot of trinomials to factor, but we don't tell you which kind you are dealing with. You will need to learn how to factor any trinomial, no matter what kind it is. Basically, you just need to use you table, or recognize it as a special case and deal with it that way. Work the following problems to practice your factoring skills.

A. Read through the "***To Factor a Polynomial***" highlight on page 334 of your text.

1: Arrange terms in proper order (highest exponent first).

2: If the first term is negative, divide by –1 and place terms in parenthesis. Now factor the expression inside the parenthesis. But don't throw away the negative sign in the final answer.

$$-x^2+2x+3 = -\left(x^2-2x-3\right)$$

3: Look for other common factors

4: Two terms: Look for difference of squares.
Three terms: Is it a perfect square; if not then use trial and error to factor.
Four terms: Look for a binomial factor- then use the grouping method.

5: Factor completely. Look at your answer. Verify there are no more factors.

6: Check by multiplying.

B. Work through the following problems: (Make your own tables)

1. $2x^5 - 8x^3 =$

2. $3x^4 - 18x^3 + 27x^2 =$

3. $y^3 + 25y =$

4. $6a^2 - 11a + 4 =$

5. $6x^3 - 12x^2 - 48x =$

6. $2ab^5 + 8ab^4 + 2ab^3 =$

7. $xy + 8x + 3y + 24 =$

Section 5.6: Solving Quadratic Equations by Factoring

This is the section where we begin to see why factoring is **sooooo** important. In this section we will learn to solve polynomial equations, especially quadratic equations.

A. Definition: A quadratic equation is said to be in standard form if it is written as follows: $ax^2 + bx + c = 0$.
   1. Notice that the equation is set equal to zero.
   2. Notice that the terms are in descending order of degree.
   3. First term is Positive.

B. A second Definition: ***<u>The Zero Property:</u>***
   1. If $a \cdot b = 0$, then either a = 0, or b = 0.
   2. Believe it or not, this property is the foundation for solving polynomial equations. We will use it a lot!!!

C. Strategy for Solving a Quadratic Equation by Factoring:
   1. Put the equation in standard form: Write it as: $ax^2 + bx + c = 0$. This first step is one of the most important and often takes the most work.
   2. Factor the quadratic equation completely!
   3. Set each factor equal to zero by using the zero property described above.
   4. Solve each of the equations created in #3 above.
   5. Check each solution.

D. Work through the following homework problems for practice. Be sure to work through the checklist given above.
   1. $(a + 6)(a - 1) = 0$

   2. $m(2m - 5)(3m - 1) = 0$

3. $a^2 - 11a + 30 = 0$

4. $100x^2 - 300x + 200 = 0$

5. $a^2 - 36 = 0$

6. $x^2 + 5x = 0$

7. $9x^2 = 12x - 4$

8. $x(12 - x) = 32$

9. $(y + 4)^2 = y + 6$

10. $9y^3 + 6y^2 - 24y = 0$

12. $x^3 + 5x^2 - 9x - 45 = 0$

Section 5.7: Solving Applications and the Pythagorean Theorem

In this section we will solve application problems that require the use of quadratic equations. Work through the following problems to see how this is done.

A. Some Practice homework problems:

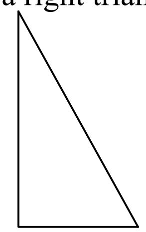
1. The product of two consecutive odd integers is 99. Find the two integers:

2. One number is 4 times another. Their product is 4 times their sum. Find the numbers.

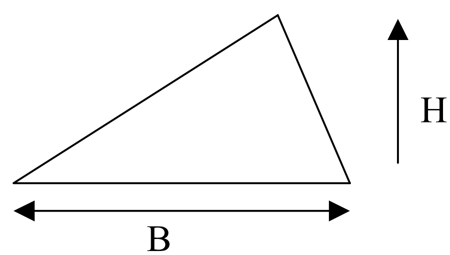
3. The length of a rectangle is 3 more than twice the width. The area is 44 square inches. Find the dimensions.

4. The hypotenuse of a right triangle is 15 inches. One of the legs is 3 inches more than the other. Find the lengths of the two legs.

Remember the Pythagorian Theorem:
In a right triangle:



5. The height of a triangle is 3 cm less than the length of the base. The area is 35 square cm. Find the height and the base.



Math 90 Lecture Notes
Chapter 6

Section 6.1: Rational Expressions

A ***Rational Expression*** is any expression that can be written in the form $\frac{P}{Q}$, where P and Q are Polynomials and Q $\neq 0$. The following are examples of Rational Expression:

$\frac{2x+3}{x}$ Numerator = _______________ , Denominator = __________

$\frac{x^2-6x+9}{x^2-4}$, Numerator = ____________ , Denominator = __________

In any rational expression we must be careful to remember that the denominator can not equal zero, since division by zero is undefined!!

A. State the restrictions on the variable in the given rational expressions: (In other words, for what values of x is the expression undefined?)

1. $\frac{x-5}{x+3}$

2. $\frac{2x-7}{x^2-3x-10}$

B. A review of the properties of working with fractions/Rational Expressions:
   1. Multiplying the numerator and denominator of a Fraction/Rational Expression by the same nonzero quantity will not change the value of the rational expression.
   2. Dividing the numerator and denominator of a Fraction/Rational Expression by the same nonzero quantity will not change the value of the rational expression. (We use this property to reduce fractions).

C. **PROBLEMS** State what values of the variable make the expression undefined then reduce the expressions to lowest terms by factoring the numerator and denominator and canceling where possible.

1. $\dfrac{3x-15}{5-x}$

2. $\dfrac{x^2-9}{x^2+5x+6}$

3. $\dfrac{10a+20}{5a^2-20}$

4. $\dfrac{2x^3+2x^2-24x}{x^3+2x^2-8x}$

Section 6.2: Multiplication and Division

In this section we will multiply and divide rational expressions. Please remember that the rules for doing this is no different than the rules for multiplying and dividing fractions that contain only numbers in the numerator and denominator.

A. An example of multiplication of a fraction involving only numbers:
   1. Method one: Multiply first – then simplify!
      $\frac{3}{4}\cdot\frac{10}{21}=$

   2. Method two: Simplify first – then multiply!
      $\frac{3}{4}\cdot\frac{10}{21}=$

   3. When working with rational expressions that contain variables, it is almost always best to simplify first and then multiply later! That means factor numerator and denominator, cancel where possible, then multiply.

B. Work through the following examples to multiply rational expressions. Remember to simplify first and multiply last.
   1. $\frac{x-5}{x+2}\cdot\frac{x^2-4}{3x-15}$

   2. $\frac{3x^4}{3x-6}\cdot\frac{x-2}{x^2}$

C. Remember that to divide rational expressions, we simply multiply by the reciprocal of the fraction we are dividing by! (Sorry about the bad English)
   1. $\frac{4}{5}\div\frac{8}{9}=\frac{4}{5}\bullet\frac{9}{8}$

D. Divide the following rational expressions:

1. $\dfrac{3x-9}{x^2-x-20} \div \dfrac{x^2+2x-15}{x^2-25}$

2. $\dfrac{4x-8}{x+2} \div \dfrac{x-2}{x^2-4}$

3. $(y^2-9) \div \dfrac{y^2-2y-3}{y^2+1}$

4: $\dfrac{a^5}{b^4} \div \dfrac{a^2}{b}$

Note: Not all problems will simplify quite as much as did the examples above.

Section 6.3: Addition, Subtraction, and Least Common Denominators

In this section we will learn how to add rational expressions. Remember that a rational expression is just a "fancy" fraction. You add these just as you would numerical fractions. You must find a common denominator before you can add or subtract. Finding the common denominator will involve factoring.

A. Some numerical Examples:

1. $\frac{2}{7}+\frac{3}{7}=$

2. $\frac{5}{10}+\frac{3}{6}=$

B. Some examples involving rational expressions with common denominators:

1. $\frac{5}{x}+\frac{7}{x}=$

2. $\frac{x}{x^2-49}+\frac{7}{x^2-49}$

3. $\frac{x-5}{x^2-4x+3}+\frac{2}{x^2-4x+3}$

4. $\frac{3-2x}{x^2-6x+8}+\frac{7-3x}{x^2-6x+8}$

5. Find the least common multiple (LCM) $a+1,\ (a-1)^2,\ a^2-1$

Section 6.4: Addition and Subtraction with Unlike Denominators

In this section we will concentrate on rational expressions that involve unlike denominators. You will need to find a common denominator before adding or subtracting these fractions. Then, as before, you will work to simplify your answer.

See page 387 for a table of rules for adding and subtracting rational expressions with different denominators.

A. Some examples involving rational expressions with different denominators:

1. $\frac{2}{x} - \frac{1}{3}$

2. $\frac{7}{3x+6} + \frac{x}{x+2}$

3. $\frac{1}{x-3} + \frac{-6}{x^2-9}$

4. $\dfrac{5}{x^2-7x+12}+\dfrac{1}{x^2-9}$

5. $\dfrac{x+4}{2x+10}-\dfrac{5}{x^2-25}$

6. $1-\dfrac{1}{x}$

7. $\dfrac{y^2}{y-3}+\dfrac{9}{3-y}$

Section 6.6: Solving Rational Equations

In working with fractions that involve rational expressions, we work to rid ourselves of the fractions involved by finding common denominators. Once we do this, the equations will look very similar to ones we have already learned how to solve in previous sections of the text.

STEPS FOR SOLVING RATIONAL EQUATIONS:

1. List the restrictions.
2. Clear the denominator
3. Solve
4. Check answer with original problem

A. Some Examples:

1. $\frac{x}{3} + \frac{5}{2} = \frac{1}{2}$

2. Solve the same problem as above, but find the common denominator first:
   $\frac{x}{3} + \frac{5}{2} = \frac{1}{2}$

3. $\frac{4}{x-5} = \frac{4}{3}$

4. $1 - \frac{3}{x} = \frac{-2}{x^2}$

5. $\dfrac{x}{x^2-9}-\dfrac{3}{x-3}=\dfrac{1}{x+3}$

6. $\dfrac{x}{x-3}+\dfrac{3}{2}=\dfrac{3}{x-3}$

1. Check your answer to this equation by plugging it into the original equation.
2. Surprised?????
3. What are the restrictions on the rational expressions contained in this

7. $\dfrac{a+4}{a^2+5a}=\dfrac{-2}{a^2-25}$

1. What restrictions are there on the variables for the rational expressions in this equation?
2. Be sure to check your answer(s).

Math 90 Lecture Notes
Chapter 7

Section 7.1: Systems of Equations and Graphing

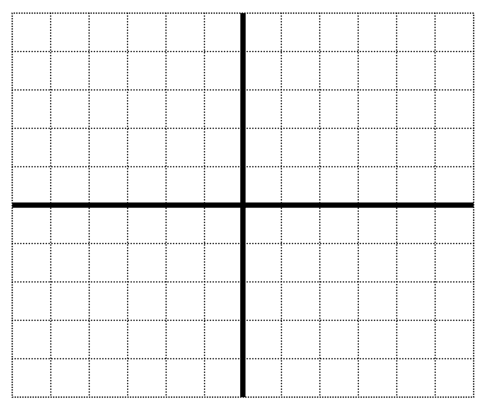
In this chapter we will learn three ways to solve what is known as a system of linear equations. In many situations, it is helpful and necessary to use more than one equation to model a real world situation. We then solve this "system" of equations to find our solution. Read the following example to see how this can work.

In this section we will also study the three types of systems. See pages 432 and 433 in the text book.

a) Consistent-Independent---> Lines that cross
b) Consistent-dependent---> Lines laying on top of each other
c) Inconsistent-independent---> Parallel lines

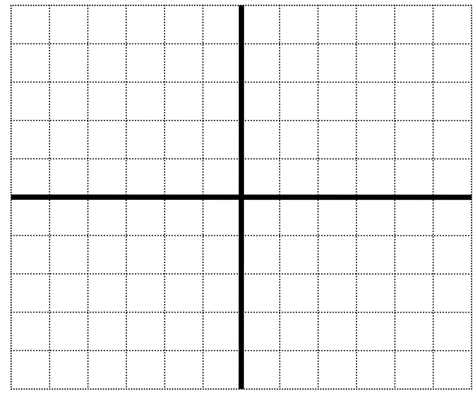
Dependent: One equation is a multiple of the other.
Consistent: A pair of equations that result in only one solution.

A. In this section, we will use the method of Graphing to solve systems of linear equations.
   1. Solve the following system by graphing:
      $x + y = 4$
      $x - y = -2$

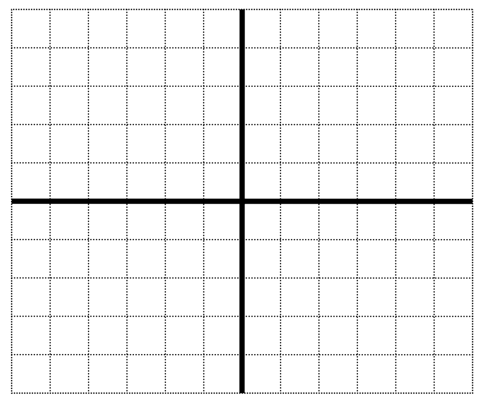


      a. Note: Since each equation we are graphing is linear, its graph will be a line.
      b. We are looking for the point where the graphs intersect. This is a point that they both share and is our solution.

2. Solve the following system of equations:
   $2x - 4y = 8$
   $2x - y = -1$



3. Solve the following system of equations:
   $y = \frac{2}{3}x + 2$ and $y = \frac{2}{3}x - 3$



4. The example given above shows that in some cases, there are no solutions to a system of equations. This occurs in cases when the lines represented by those equations that are parallel.

5. It is also possible for a system of equations to have an infinite number of solutions. Look at the following system to see how this can happen. Solve the following: $2x + y = 4$ and $4x + 2y = 8$

B: **DO THESE PROBLEMS**

Solve the sets of simultaneous equations, in the following manner, by graphing.

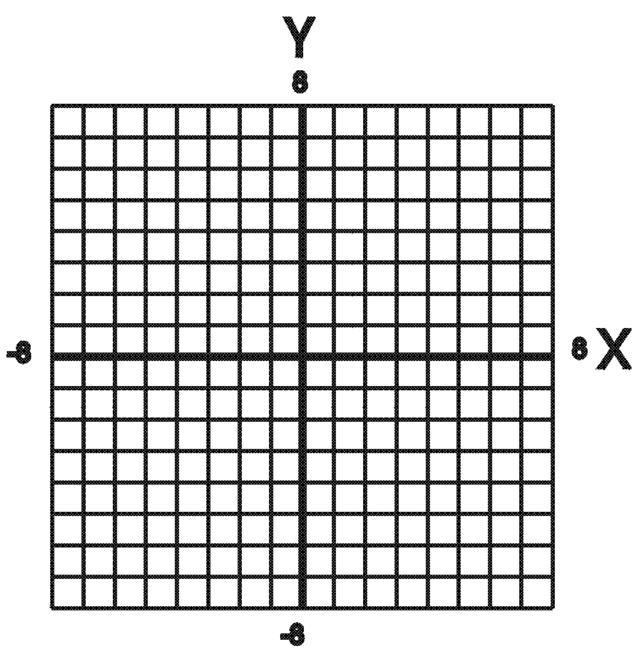
1. For each value of x given, compute y and enter it in the table.
2. Plot the coordinates given in the tables.
3. The resulting points should all lie on a straight line.
4. Do this for each equation.
5. Plot the two equations on the left on the left grid; plot the two equations on the right on the right grid.

$y = x + 4$

X	Y
0	
-2	
+2	
-4	
+4	

$y = -2x - 4$

X	Y
0	
-2	
2	
-4	

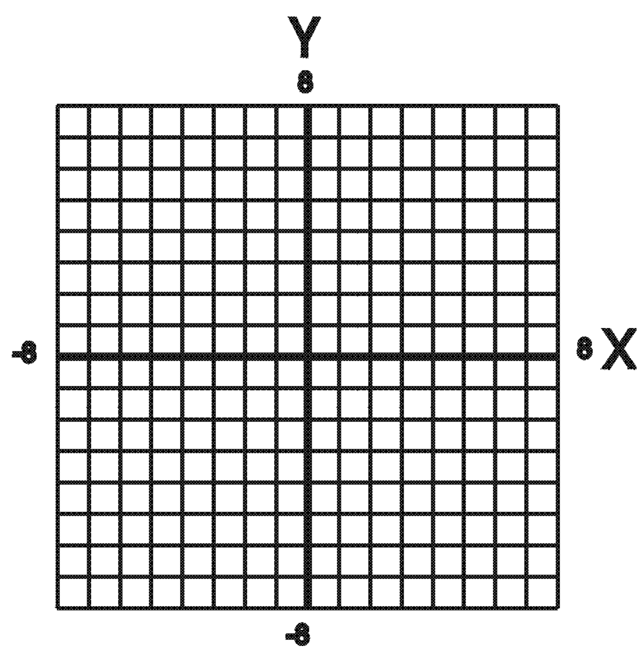


$2x + y = 8$

X	Y
0	
2	
3	
4	

$x - y = 7$

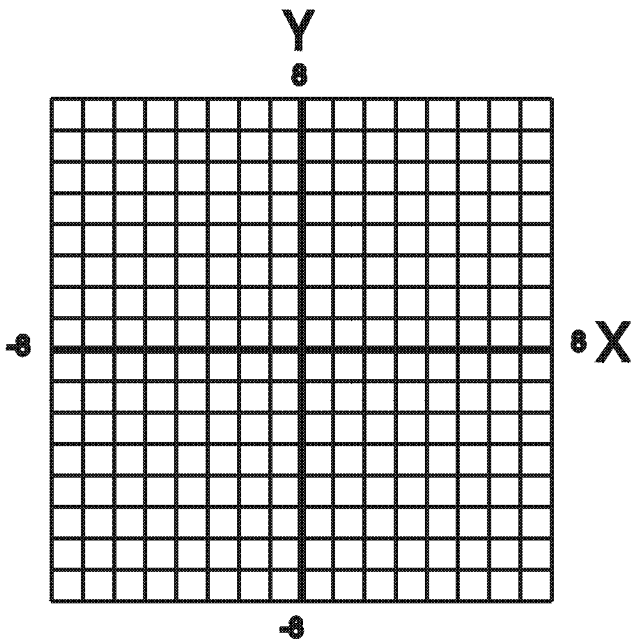
X	Y
0	
2	
3	
4	
-1	

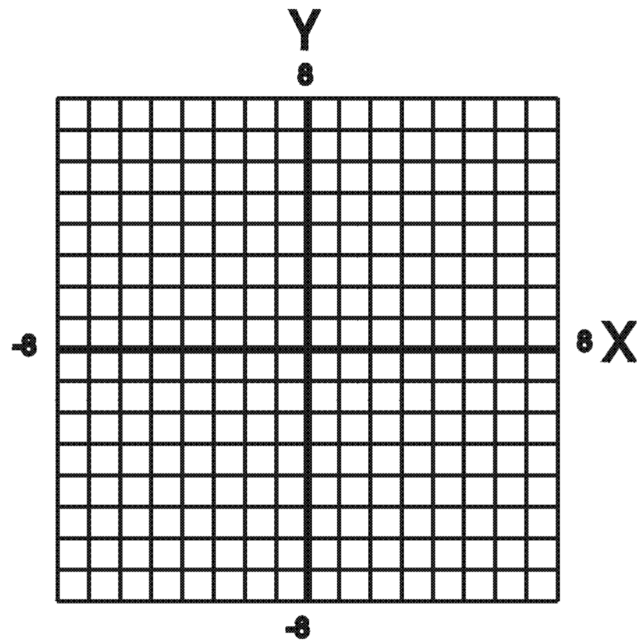


In each of the following problems, graph both equations (by any method) on a single grid. If there is a common solution to the two equations, the two straight lines will intersect and the point of intersection is the solution. If the lines intersect, write the point of intersection as an ordered pair. If the lines do not intersect, there is no solution. If the lines are coincident (lie one on top of the other) there are an infinite number of solutions.

4. $4x - 20 = 5y$
$8x - 10y = 12$

5. $y = \frac{1}{3}x + 2$
$3y = x - 21$





Section 7.2: Systems of Equations and Substitution

The method of substitution for solving systems of linear equations works for any linear system, but works best if one of the equations in the system can easily be solved for one of the variables. What we look for in particular is a variable in either equation that has a coefficient of 1. This makes the substitution method very easy to use. Work through the following examples to see how easy this method is to use. Remember that we are trying to find a point where the two lines intersect, and a point has both an x and a y coordinate.

A. Some sample problems: Solve using the method of Substitution.

1. $2x + y = 1$
   $x - 5y = 17$

2. $2x - 4y = -4$
   $x + 2y = 8$

3. $7x + 6y = -9$
   $y = -2x + 1$

B. Remember that not all systems of linear equations have one unique solution. What are the other cases?

   1. If the lines are parallel, how many solutions will you find? ______

   2. If the lines coincide, how many solutions will you find? _______ .

C. Some rules for solving systems of linear equations

   5. If possible, remove by factoring, any constant multiplier that may be in the equations. This action will not change the equality but will make the equation simpler.
   6. Re-arrange the equations such that both are in the same form. For instance place all terms on the left with zero on the right.
   7. If the two equations are identical, then these equations lie one on top of the other and there are an infinite number of solutions.
   8. If the variable terms are identical but the constants different then these are parallel lines and there is no solution.
   9. If options 3 and 4 are both false then there is one solution.

D. Work through the following examples to see what happens when you encounter one of the situations described above:

   1. $2x + 6y = -18$ How many solutions? _____________
      $x + 3y = -9$

   2. $10x + 2y = -6$ How many solutions? _____________
      $y = -5x$

E. Some examples of application problems:

1. Find two numbers such that the sum is 76 and the difference is 12.

2. Two angles are supplementary. One angle is 8 degrees less than three times the other. Find the measure of each angle.

3. The state of Wyoming is a rectangle with a perimeter of 1280 miles. The width is 90 miles less than the length. Find the length and the width.

Section 7.3: Systems of Equations and Elimination

In this section we will learn another method for solving systems of equations called the Elimination Method. In this method, you want to ultimately add the two equations together and eliminate one of the variables. Work through the following examples to see how this is done.

A. The Rational behind this method: Note the following:

If A = B and
C = D then it is true that: A+C = B+D

Because of this fact, we can always add two equations together to get another true equation. We will use this fact when we use the elimination method for solving systems of equations.

B. Some sample problems: Solve the following using the Elimination Method. But before solving these equations let us examine them to determine what kind of solution we can expect.

1. $x - y = 1$
   $-x - y = -7$

2. $x - y = 4$
   $2x + y = 8$

3. $-x + 10y = 1$
   $-5x + 15y = -9$

4. $2x + 9y = 2$
   $5x + 3y = -8$

C. Once again remember that not all systems of equations have a unique solution. Follow the next two examples to see what happens when we get no solution, or an infinite number of solutions.

1. $8x - 2y = 2$
   $4x - y = 2$ How many solutions does this system have?

2. $$\frac{-1}{3}x - \frac{1}{2}y = -\frac{2}{3}$$
   $$-\frac{2}{3}x - y = -\frac{4}{3}$$
   How many solutions does this system have?

Section 7.4: More Applications using Systems

In this section we will practice solving application problems using systems of equations. The most important part of this process is setting up the system that models the situation. You can use either the substitution method or the elimination method to solve the system that you set up.

A. Sample application problems:

1. In winning the 2000 conference finals, the Los Angeles Lakers scored 69 of their points on a combination of 31 two and three pointers. How many of each type of shot did they make?

2. Filmworks charges \$1.75 for a role of 24-exposure film and \$2.25 for a 36-exposure roll of film. Stu bought 19 rolls of film for a total of \$39.25. How many rolls of each type did he buy?

3. From November 2 through January 3, the Bronx Zoo charges \$6.00 for adults and \$3.00 for children and seniors. One December day, a total of \$1554 was collected from 394 admissions. How many adult and how many children/senior admissions were there?

4. Sunflower seed is worth \$1.00 per pound and rolled oats are worth \$1.35 per pound. How much of each would you use to make 50 pounds of a mixture worth \$1.14 per pound.

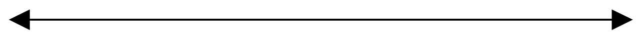
5. Clear Shine window cleaner is 12% alcohol and Sunstream window cleaner is 30% alcohol. How many ounces of each should be used to make 90 ounces of a cleaner that is 20% alcohol?

Section 7.5: Linear Inequalities in Two Variables

In this section we will learn how to find solutions of linear inequalities in two variables.

A. An example of an inequality in one variable. Solve and graph the solution set for the following inequality:

$$3(x+5) \le 22$$

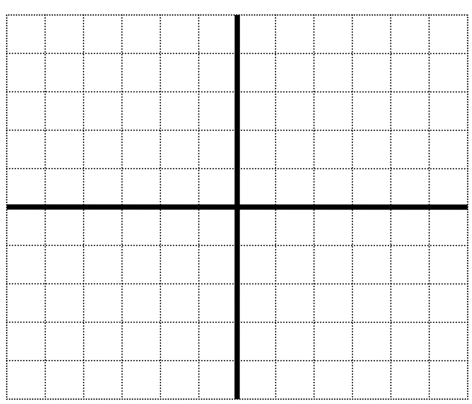


Note: There are an infinite number of solutions to this inequality. Because of this we need to describe our solutions and/or graph them as it would be impossible to list them all.

B. Find and graph as many ordered pair solutions to the following inequality.

$$y \le x + 3$$

X	Y



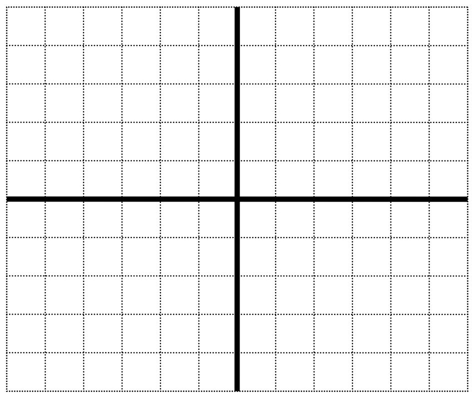
C. What do you notice about the boundary for the solutions to this inequality in two variables?

1. ____

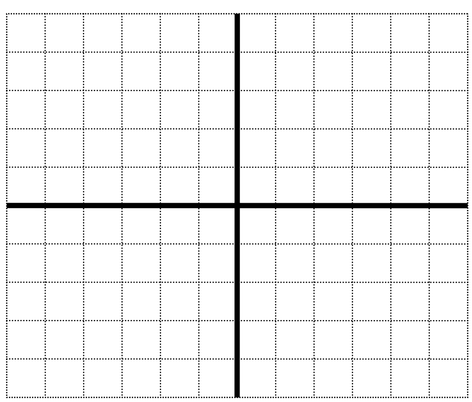
2. ____

D. Graph the solution set for the following inequalities in two variables:

1. $y \le 3x + 2$



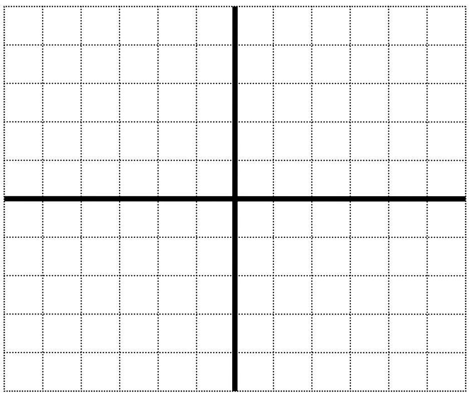
2. $5x + 4y \ge 20$



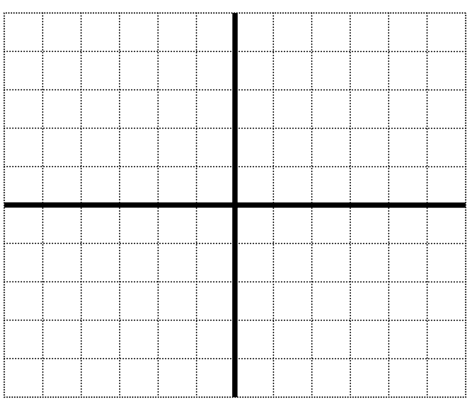
Remember that when you are graphing the solution set to an inequality in two variables that you must first graph the "Boundary Line". The boundary line will be solid when: ______________________ , and the boundary line will be dashed or dotted when: _______________________ .

E. The next two examples will give you an idea of possible applications to this process of writing inequalities in two variables.

1. A soccer team that was sponsored by “Pizza Hut” won their division. As a reward, the Pizza Hut restaurant told the manager that the team could celebrate their awards ceremony at their restaurant and that they could order as many pizzas as they wanted as long as the total cost did not exceed \$180. Small pizzas cost \$10 each and large pizzas cost \$18 each. Graph the solution set for this situation.



2. A certain elevator in an old historic building in San Diego has a weight capacity of 1050 pounds. Research has shown that on the average, children weigh 75 pounds and adults weigh on the average 150 pounds. What combinations of adults and children would cause this elevator to be overloaded? Graph this solution set.



CHAPTER 8

Section 8.1: Introduction to Square Roots and Radical Expressions

In this chapter we will learn how to work with radical expressions such as square roots. A good background in this type of algebra will help us a great deal as we work with solving polynomial equations that cannot be factored. We will learn how to do this in chapter 9.

In general the square root of any positive number has two solutions: a positive solution and a negative solution. But, by convention, when a radical expression is presented by itself (that is not part of an equation) then we take only the positive solution and discard the negative solution. This positive answer is called the 'principle value'. The reason why this is done is related to the mathematics of functions and is not covered in this course. On the other hand, when radicals are presented as part of an equation then there may be two solutions.

A. A Square Root: The square root of a number "x" is a number "y" such that $y^2 = x$. Some examples of square roots are as follows:

1. $\sqrt{25} =$

2. $\sqrt{9} =$

3. $\sqrt{47} =$

B. Radical Expressions: A radical expression is an algebraic expression that contains at least one radical sign. Some examples are as follows:

1. $\sqrt{29} =$

2. $3x + \sqrt{3x+5}$

3. $\sqrt{\dfrac{5x^2 - 2x + 3}{6x - 5}}$

Note that the expression under the radical is called the "**radicand**"

C. Irrational Numbers: Irrational numbers are numbers that cannot be written as fractions with integers in the numerator and denominator, or decimals that terminate or repeat. One of the most famous irrational numbers is "π". "Pi" is the ratio of the circumference of a circle to its diameter. In this chapter, most of the irrational numbers we will come across will be square roots of numbers that are not perfect squares. Identify the following as rational or irrational numbers.

1. $\sqrt{13} =$

2. $\sqrt{36} =$

3. $\sqrt{8} =$

4. $\sqrt{144} =$

D. Square roots and "Absolute Value":

For any real number "A", $\sqrt{A^2} = |A|$. Copy an example from your instructor that illustrates why the absolute value must be used in this type of situation:

E. Simplifying rational expressions containing variables: Work through the following examples: Assume all variables represent non-negative values.

1. $\sqrt{x^2} =$

2. $\sqrt{25y^2} =$

3. $\sqrt{(7z)^2} =$

4. $\sqrt{49x^2y^2z^2} =$

F. Approximating square roots that are irrational: (Use you calculators to give an approximation to the following square roots that are not rational. Round each to the nearest hundredth)

1. $\sqrt{7} \approx$

2. $\sqrt{119} \approx$

3. $\sqrt{27} \approx$

Note: Please understand the difference between simplifying a square root and approximating a square root. What you just did in the three problems above was approximation, not simplification!

Section 8.2: Multiplying and Simplifying Radical Expressions

Not all square roots are square roots for perfect squares as you saw in the last section. In this section you will learn how to simplify non-perfect square roots without approximating them.

A. Multiplying Square Roots: The following is the rule for multiplying square roots.

$\sqrt{A} \cdot \sqrt{B} = \sqrt{A \cdot B}$ (Note that the opposite is true as well)

$\sqrt{A \cdot B} = \sqrt{A} \cdot \sqrt{B}$ (These rules will serve us well)

B. Multiply the following square roots but do not simplify at this time:

1. $\sqrt{7} \cdot \sqrt{3} =$
2. $\sqrt{6} \cdot \sqrt{10} =$
3. $\sqrt{3} \cdot \sqrt{5} \cdot \sqrt{6} =$

C. Simplifying a radical expression: When you want to simplify a radical that is not a perfect square you will factor it into a special product. Record what you instructor shows you about this important skill.

D. Simplify the following examples:

1. $\sqrt{20} =$
2. $\sqrt{75} =$
3. $\sqrt{25x} =$
4. $\sqrt{18x^2} =$
5. $\sqrt{x^8} =$
6. $\sqrt{y^{14}} =$
7. $\sqrt{z^7} =$
8. $\sqrt{x^{11}} =$

9. $\sqrt{75x^2} =$

10. $\sqrt{27x^5} =$

11. $\sqrt{28t^6} =$

12. $\sqrt{50x^3} =$

E. Multiply the following radical expressions and then simplify:

1. $\sqrt{3} \cdot \sqrt{6} =$

2. $\sqrt{6} \cdot \sqrt{8} =$

3. $\sqrt{15m^7} \cdot \sqrt{5m} =$

4. $\sqrt{xy} \cdot \sqrt{xz} =$

5. $\sqrt{6a} \cdot \sqrt{2a} =$

6. $\sqrt{10xy^2} \cdot \sqrt{5x^2y^3} =$

Section 8.3: Quotients Involving Square Roots

A. We have already learned to simplify radicals in sections 8.1 and 8.2. In this section we will learn how to remove radicals from the denominator of a fraction.

1. Removing the radical from the denominator sometimes makes the expression easier to evaluate.
   a. Estimate the following: $\dfrac{3}{\sqrt{5}}$
   b. Rationalize the denominator: (This means that we will remove the radical, or irrational number from the denominator of the fraction).
   c. Now estimate the result from "b" above:

B. The Quotient Rule for Square Roots:

$$\sqrt{\frac{A}{B}} = \frac{\sqrt{A}}{\sqrt{B}} \qquad \text{The opposite of this is true as well: } \frac{\sqrt{A}}{\sqrt{B}} = \sqrt{\frac{A}{B}}$$

We will use these rules to help us simplify radical expressions that involve quotients: Work through the examples below to find out how.

C. Note the following:

1. $\sqrt{5} \cdot \sqrt{5} =$

2. $\sqrt{7} \cdot \sqrt{7} =$

3. $\sqrt{11} \cdot \sqrt{11} =$

D. Rationalize the denominator or simplify each of the following expressions.

1. $\sqrt{\dfrac{1}{6}}$

2. $\sqrt{\dfrac{32}{5}}$

3. $\sqrt{\dfrac{12}{3}}$

4. $\sqrt{\dfrac{21}{7}}$

5. $\dfrac{8\sqrt{50}}{16\sqrt{2}}$

6. $\sqrt{\dfrac{9x^2y^2}{3}}$

7. $\dfrac{5\sqrt{27x^3y^2}}{2}$

Section 8.4: More Operations with Radicals

Adding radical expression is very similar to adding "like terms" in polynomials.

A. Some examples of polynomial addition and subtraction:

1. $(2x^2 + 3x - 5) + (7x^2 - 6x + 11) =$

2. $(3x^3 - 7x^2 + 5x - 7) - (6x^2 - 5x + 4) =$

B. Some examples of addition of radical expressions. In these examples the radicals are treated somewhat like a variable (e.g. x or y) because they cannot be further simplified.

1. $3\sqrt{2} + 5\sqrt{2} =$

   So, just as $3x + 5x = 8x$ then $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$

2. $7\sqrt{3} + 5\sqrt{10} - 3\sqrt{3} + 2\sqrt{10} =$

C. Some examples that are a little more complicated. To solve these problems we must simplify the radial expressions.

1. $3\sqrt{12} + 5\sqrt{48} =$

2. $4\sqrt{18} + \sqrt{32} - \sqrt{2} =$

3. $2\sqrt{27x^2} - x\sqrt{48} =$

4. $9\sqrt{24x^3y^2} - 5x\sqrt{54xy^2} =$

5. $8\sqrt{72x^2} - x\sqrt{8} =$

6. $\dfrac{8+\sqrt{48}}{8} =$

7. $\dfrac{-12+\sqrt{20}}{6} =$

D. More examples that involve multiplication:
Multiply the following expressions and simplify when possible:

1. $\sqrt{2}\cdot\sqrt{5} =$

2. $\sqrt{6}\cdot\sqrt{3} =$

3. $\left(3\sqrt{2}\right)\left(4\sqrt{5}\right) =$

4. $\left(7\sqrt{6}\right)\left(3\sqrt{2}\right) =$

5. $\sqrt{3}\left(\sqrt{5}+2\right) =$

6. $\sqrt{5}\left(\sqrt{7}-\sqrt{5}\right) =$

7. $\left(3+\sqrt{2}\right)\left(2-\sqrt{6}\right) =$

8. $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right) =$

9. $\left(5-\sqrt{2}\right)^2 =$

10. $\left(2\sqrt{x}+4\right)\left(3\sqrt{x}+2\right) =$

Section 8.5: Radical Equations

In this section we will learn how to solve equations that involve Radical expressions involving square roots. We will accomplish this by using a technique that involves squaring both sides of the equation with which we are working. However, we need to be especially careful in this process because we sometimes arrive at solutions that do not work in our original equation. Note the following illustration to see what can often happen:

1. Take the following equation: x = 5
2. Square both sides of this equation: _________________
3. Solve this new equation: _________________________
4. How many solutions did the original equation have?
5. How many solutions did the new equation have after we squared both sides of the equation?
6. What happened?

Obviously squaring can make 'funny' things happen to an answer.

A. Solve the following equations and remember to check your results for **"Extraneous"** solutions. Before we actually solve anything let's take a look at these problems and identify those problems where two answers are likely to appear.

   Indentify the problems below that may have two answers_____________

   1. $\sqrt{x+5} = 7$

   2. $\sqrt{2x-5} = 7$

   3. $\sqrt{x+7} = -4$

4. $\sqrt{x+3} = x-3$

5. $\sqrt{y-4} = y-6$

6. $2\sqrt{a} = 12$

7. $\sqrt{2x+4} = 9$

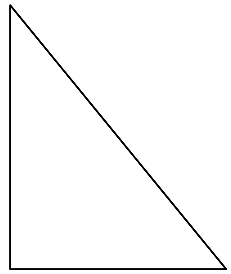
8. $\sqrt{2x+5} = \sqrt{3x+7}$

Section 8.6: Applications Using Right Triangles

In this section we will use "Pythagorean Theorem" to solve problems that deal with right triangles.

A. Identify the following for the given right triangle:

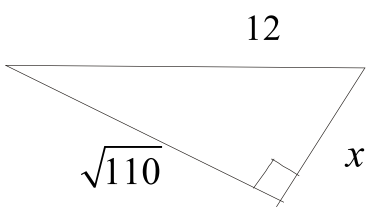
1. Legs
2. Hypotenuse
3. Right angle



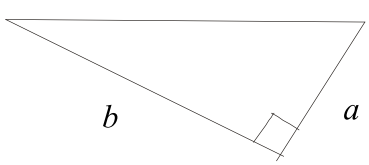
B. Record the formula for the Pythagorean Theorem below:

C. DO THESE PROBLEMS:

1. Find the length of the third side (x). If the answer is not a whole number use radical notation.



For each of the two problems below, in a right triangle, find the length of the side not given.

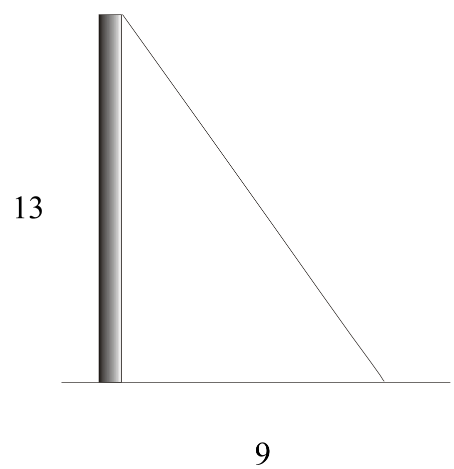


2. $a = 12\ ; b = 5 ; c = ?$

3. $b = 1 ; c = \sqrt{5}\ ; a = ?$

4. *MASONRY.* Find the length of a diagonal of a square tile with 4 cm sides.

5. How long must a guy wire be to reach from the top of a 13-m telephone pole to a point on the ground 9 m from the foot of the pole.



6. A soccer field is 100 yd long and 50 yd wide. What is the length of the diagonal?

Chapter 9

Section 9.1: More Quadratic Equations

In this section we will learn some more ways to solve quadratic equations. At this point in our study, you can only solve quadratic equations by factoring. However, many of the quadratic equations that you will encounter in mathematics and in subjects that require mathematics will not be factorable. How then will we solve those types of quadratics? We will learn three new techniques in the next three sections that will give you this ability.

A. Square Root Property for Equations:
   If $a^2 = b$, then $a = \pm\sqrt{b}$
   Using this rule for quadratics, we can solve equations similar to the following.

B. Some sample problems that utilize the "Square Root Property".
   1. $x^2 = 25$

How we would have done #1 before:

   2. $x^2 = 11$

   3. $5x^2 = 100$

   4. $(a + 3)^2 = 16$

   5. $(2x + 5)^2 = 49$

6. $(2x - 3)^2 = 7$

7. $\left(a - \dfrac{3}{7}\right)^2 = \dfrac{18}{49}$

8. The square of the sum of twice a number and 3 is 25. Find the number. (Actually, there are two different solutions)

Section 9.2: Solving Quadratic Equations: Completing the Square

In this section we will learn a new method for solving quadratic equations called completing the square. This technique will prove very helpful in other areas of mathematics as well.

Remember that we can solve the following types of equations by simply taking the square root of both sides:

$$(2x+5)^2 = 27$$

When we are faced with solving a quadratic equation that cannot be factored, we can solve it by turning the equation into one that looks very much like the one you see above. Work through the following problems to see how this works. Solve the following:

1. $x^2 + 10x - 4 = 0$

2. $x^2 + 6x + 8 = 0$

3. $x^2 - 10x + 14 = 0$

(Note: When completing the square, the leading coefficient must be equal to one)

4. $3x^2 + 24x - 3 = 0$

Section 9.3 The Quadratic Formula and Applications

In this section we will learn how to solve quadratic equations by using the quadratic formula. The quadratic formula is simply a formula which allows us to solve any quadratic equation, assuming it is in standard form.

Remember that a quadratic equation is in standard form if it looks like the following: $ax^2 + bx + c = 0$, and "a" is a positive number.

The quadratic formula, which you will use shortly, is found by completing the square in the above "standard form" quadratic equation: Do this below.

A. ***The Quadratic Formula***: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ The quadratic formula will help you to solve any quadratic equation you come up against. For any particular equation however, the quadratic formula may be "too much work" and you might find it easier to solve the equation using some of the other techniques we have already discussed. Study each problem carefully before you decide which method to use!

1. Solve the following using the quadratic equation: $5x^2 - 3 = 14x$

2. Solve the following using the quadratic equation: $x^2 - 7x = 8$

3. Solve the following using the quadratic equation: $y^2 + 5y + 3 = 0$

4. Solve the following using the quadratic formula: $x^2 + 6x + 7 = 0$

5. The world record for free-fall to the ground without a parachute by a woman is 175 feet and is held by Kitty O'Neill. Approximately how long did the fall take? Use the following formula for which "t" is the time in seconds and "s" is the distance in feet the person fell. ($s = 16t^2$)

6. The length of a rectangle is 3 meters greater than the width. The area is 70 square meters. Find the length and the width.

THE END

YOU DID IT !

CONGRATULATIONS TO YOU!