Section 5.1: Introduction to Factoring

This is the chapter in which we learn the very important skill of factoring polynomials, especially trinomials. Please remember that when we ask you to factor something, we are asking you to ***write it as a product***. We start out by learning how to factor out the greatest common factor from an expression.

A. A Little Review: Multiply the following:

1. $7(2x + 5)$

2. $2x(5x^2 + 3)$

3. $3x^2(5x - 7)$

4. $3x^3(2x^2 - 3x + 5)$

B. Factoring out the greatest common factor is just the opposite of what you did in problems #1 – 4 above. We start with the answer and work backwards, writing it as a product. Factor out the greatest common factor for the following problems.

1. $14x + 21$

2. $7x^3 - 4x^2$

3. $16x^4 - 20x^2 - 16x$

4. $5ab^2 + 10a^2b^2 + 15a^2b$

5. $49xy - 21x^2y^2 + 35x^3y^3$

C. **Factoring by Grouping**: ***We will use this technique when we have 4 terms that we want to factor***. Sometimes polynomials with four terms contain binomial factors. In these cases, we group the first two terms and factor out the greatest common factor from them, then we group the second two terms and factor out the greatest common factor from them. Hopefully at this point, we have a common factor that will allow us to factor the original expression. Work through the following examples.

These types of problems are best manipulated mentally before writing anything down. Look at this first problem- here we see a common x in the first two terms. If we factor out the x then we are left with y+2. Now look at the second pair of terms. Here we see a common 4 and, if we factor that out, we are left with y+2 for a factor of the second two terms. So x+2 is the common binomial factor of the this four term polynomial.

1. $xy + 2x + 4y + 8$

2. $3ax + 21x - a - 7$

3. $x^3 - 5x^2 - 4x + 20$

4. $8x^3 - 12x^2 + 14x - 21$

Section 5.2: Factoring Trinomials of the type $x^2 + bx + c$

In this section of the text, we learn how to factor trinomials. This is one of the most important parts of this course! The ability to factor will make or break you in the next algebra course that you take. The text shows you a method of trial and error for factoring.

See pages 80 and 84 of this book for guidance on determining the algebraic signs in the factors.

Successful factoring requires PRACTICE, PRACTICE, PRACTICE!!!

Plan to spend some time on this concept – it will pay off "***BIG TIME***" later in this course and in the future!

A. A review of multiplication using the table: Multiply the following using the table:

1. $(x + 7)(x – 5)$

2. $(x + 2)(x + 6)$

B. Factor the following:

FACTORS OF 12

CHECK BY MULTIPLYING

1. $x^2 + x – 12$

i. Determine the signs
ii. List the factors of 12
iii. Enter the two selected factors
iv. Check ans.

$$x = (x \qquad)(x \qquad)$$

2. $x^2 - 5x - 24$

FACTORS OF 24

CHECK BY MULTIPLYING

$x = (x \qquad)(x \qquad)$

3. $x^2 + 8x + 12$

FACTORS OF 12

CHECK BY MULTIPLYING

$x = (x \qquad)(x \qquad)$

4. $x^2 + x - 42$

FACTORS OF 42

CHECK BY MULTIPLYING

$x = (x \qquad)(x \qquad)$

5: Factor Completely: $x^4 - 11x^3 + 24x^2$ (Make your own table)

6: Factor Completely: $3x^3 - 3x^2 - 18x$ (Make your own table)

7: $6x - 72 + x^2$ (First put in standard form, make own table)

8: $11 - 3w + w^2$ (A polynomial that will not factor is called "***Prime***")

GET IT RIGHT
FACTORING

$$X^2 + BX + C$$

1. REARRAGE TERMS IN PROPER ORDER: $X^2 + X - 42$

2. PLACE THE PARENTHESIS: $X^2 + X - 42 = (\qquad)(\qquad)$

3. PUT THE X'S IN PLACE: $X^2 + X - 42 = (\ X \qquad)(\ X \qquad)$

4. NOW THE AGEBRAIC SIGNS. THE SIGNS CAN BE FOUND BY EXAMINING THE QUADRATIC. **THERE ARE ONLY FOUR CASES.**

$$X^2 + BX + C = (X + a)(x + b)$$
$$\text{P}\circ\text{P} = \text{P}\circ\text{P}$$

$$X^2 + BX - C = (X + a)(x - b)$$
$$\text{P}_{\text{LU}}\text{M} = \text{P}_{\text{LU}}\text{M}$$

$$X^2 - BX + C = (X - a)(x - b)$$
$$\text{M}\circ\text{P} = \text{M}\circ\text{M}$$

$$X^2 - BX - C = (X - a)(x + b)$$
$$\text{M}\circ\text{M} = \text{M}\circ\text{P}$$

In the above figure the P's stand for Plus and the M for Minus. Imagine PoP (dad) sitting on a couch with a soda PoP in one hand, a bowl of PluMs in the other and MoM standing nearby with a MoP ready to clean up the mess.

In this case the signs are Plus and Minus: $X^2 + X - 42 = (\ X + \qquad)(\ X - \qquad)$

5. Now the numbers. The product of the two numbers must equal the constant in the quadratic and the sum must be the coefficient of middle term . So- on a piece of paper, list the prime factors of the constant and include 1 as a factor. For 42 these factors are 1,2,3,7. From these values make all possible pairs as shown at the right.

$1, 42$
$2, 21$
$3, 14$
$6, 7$

6. The correct factors are 6 and 7 since the product is 42 and the difference is 1.

$$X^2 + X - 42 = (\ X + \ 7\ )(\ X - \ 6\ )$$

7. The final , most important step, is to check your answer by multiplication!

Section 5.3: Factoring Trinomials of the Type $ax^2 + bx + c$

In this section we practice how to factor trinomials that have a leading coefficient other than one. These usually take a little more time and effort to figure out. We will also work on factoring trinomials that require us to factor out the greatest common factor first, before we factor the trinomial. Learn these skills well!!!

There are two ways to approach problems like these:

a) Trial and error: Try all combinations of the factors of the first and last terms. Can be very time consuming but if you stick with it you will eventually arrive at the correct answer.

b) Factor by grouping approach: This is much faster but requires that you remember the various steps.

A. Some practice problems:

1. $2x^2 + 5x + 3$

2. $3x^2 - 2x - 5$

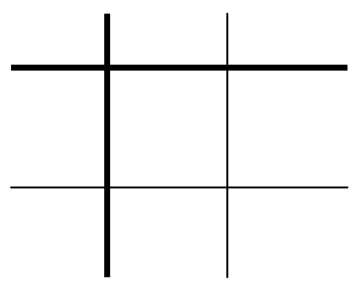
3. $6x^2 - 13x + 6$

4. $4x^2 + 12xy + 9y^2$ (Make your own table for this one)

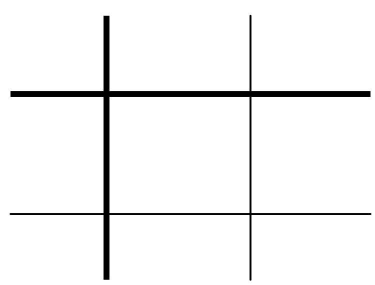
5. $4x^2 - 4x - 15$ (Make your own table for this one)

B. Factoring problems where you need to factor out the greatest common factor first – some examples:

1. $6x^2 - 51x + 63$



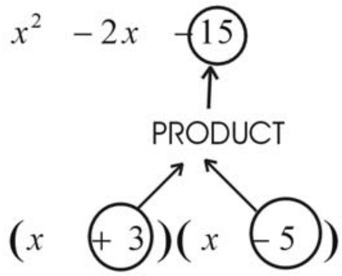
2. $18a^2 + 48a + 32$

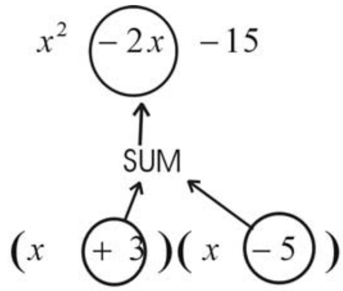


3. $10x^4 + 7x^3 - 12x^2$

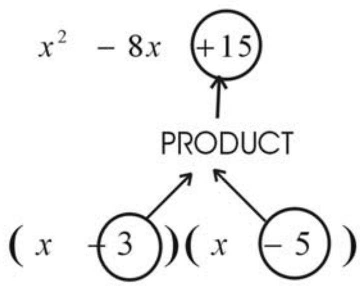
4. $6x^3 + 19x^2 + 10x$

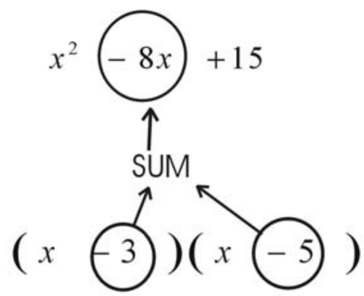
GETTING IT RIGHT
SIGNS IN THE FACTORS



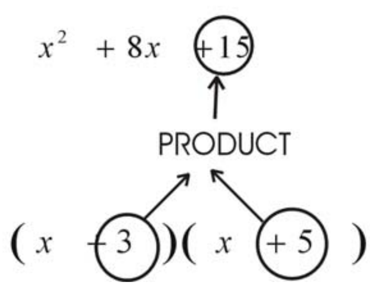


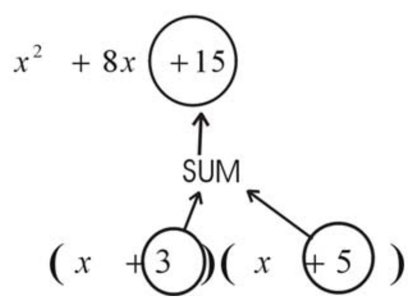
Look at the sign of the last term in the quadratic. If it is negative then one of the factors must be POSITIVE; the other NEGATIVE. This means the sign of the middle term can be either PLUS or MINUS depending on the size of the constants in the factors.





If the sign of the last term in the quadratic is PLUS then the signs in the factors must both be plus **OR** both must be negative. If the sign of the middle term in the quadratic is plus then both factors are plus (bottom figure); if the sign of the middle term in the quadratic is minus then both factors are minus (top figure).





Section 5.4: Factoring Perfect-Square Trinomials and Diff. of Squares

In this section we introduce you to a special case in factoring known as the difference between two squares. We have already looked at this subject back in Section 4.5. These are very easy to factor once you recognize them. We will also look at factoring perfect-square trinomials.

A. A quick review of multiplication - Multiply the following:
   1. $(x - 5)(x + 5) =$

   2. $(2x - 7)(2x + 7) =$

   3. $(3x + 2)(3x - 2) =$

   4. Do you remember the pattern in the 3 products given above?

B. Using the pattern you see above, how would you factor the following? You won't need the table to factor these special cases, but you could use it if you wanted to do so.
   1. $x^2 - 49 =$

   2. $y^2 - 64 =$

   3. $9x^2 - 36 =$

4. $81x^2 - 1 =$

5. $4w^2 - 25 =$

C. Factoring Perfect Squares: Use the table to factor the following:

1. $x^2 + 6x + 9 =$

2. $m^2 + 12m + 36 =$

3. $25x^2 + 10xy + y^2 =$ (Make your own table here)

$$9x^2 - 49$$

NEG

PERF SQUARE TERM

PERF SQUARE TERM

TNT

TNT BLOWS UP-IT CAN BE FACTORED AND EQUALS (3X + 7)(3X - 7)

$$9x^2 + 49$$

POS

PERF SQUARE TERM

PERF SQUARE TERM

TPT

TPT DOES NOTHING- P MEANS PHIZZLE THERE ARE NO FACTORS !

Section 5.5: Factoring: A General Strategy

In this section of the text we simply give you a lot of trinomials to factor, but we don't tell you which kind you are dealing with. You will need to learn how to factor any trinomial, no matter what kind it is. Basically, you just need to use you table, or recognize it as a special case and deal with it that way. Work the following problems to practice your factoring skills.

A. Read through the "***To Factor a Polynomial***" highlight on page 334 of your text.

1: Arrange terms in proper order (highest exponent first).

2: If the first term is negative, divide by –1 and place terms in parenthesis. Now factor the expression inside the parenthesis. But don't throw away the negative sign in the final answer.

$$-x^2+2x+3 \ = -\left(x^2-2x-3\right)$$

3: Look for other common factors

4: Two terms: Look for difference of squares.

Three terms: Is it a perfect square; if not then use trial and error to factor.

Four terms: Look for a binomial factor- then use the grouping method.

5: Factor completely. Look at your answer. Verify there are no more factors.

6: Check by multiplying.

B. Work through the following problems: (Make your own tables)

1. $2x^5 - 8x^3 =$

2. $3x^4 - 18x^3 + 27x^2 =$

3. $y^3 + 25y =$

4. $6a^2 - 11a + 4 =$

5. $6x^3 - 12x^2 - 48x =$

6. $2ab^5 + 8ab^4 + 2ab^3 =$

7. $xy + 8x + 3y + 24 =$

Section 5.6: Solving Quadratic Equations by Factoring

This is the section where we begin to see why factoring is **sooooo** important. In this section we will learn to solve polynomial equations, especially quadratic equations.

A. Definition: A quadratic equation is said to be in standard form if it is written as follows: $ax^2 + bx + c = 0$.
   1. Notice that the equation is set equal to zero.
   2. Notice that the terms are in descending order of degree.
   3. First term is Positive.

B. A second Definition: ***The Zero Property:***
   1. If $a \cdot b = 0$, then either a = 0, or b = 0.
   2. Believe it or not, this property is the foundation for solving polynomial equations. We will use it a lot!!!

C. Strategy for Solving a Quadratic Equation by Factoring:
   1. Put the equation in standard form: Write it as: $ax^2 + bx + c = 0$. This first step is one of the most important and often takes the most work.
   2. Factor the quadratic equation completely!
   3. Set each factor equal to zero by using the zero property described above.
   4. Solve each of the equations created in #3 above.
   5. Check each solution.

D. Work through the following homework problems for practice. Be sure to work through the checklist given above.
   1. $(a + 6)(a - 1) = 0$

   2. $m(2m - 5)(3m - 1) = 0$

3. $a^2 - 11a + 30 = 0$

4. $100x^2 - 300x + 200 = 0$

5. $a^2 - 36 = 0$

6. $x^2 + 5x = 0$

7. $9x^2 = 12x - 4$

8. $x(12 - x) = 32$

9. $(y + 4)^2 = y + 6$

10. $9y^3 + 6y^2 - 24y = 0$

12. $x^3 + 5x^2 - 9x - 45 = 0$

Section 5.7: Solving Applications and the Pythagorean Theorem

In this section we will solve application problems that require the use of quadratic equations. Work through the following problems to see how this is done.

A. Some Practice homework problems:

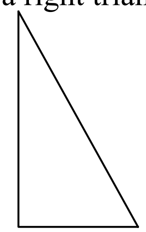
1. The product of two consecutive odd integers is 99. Find the two integers:

2. One number is 4 times another. Their product is 4 times their sum. Find the numbers.

3. The length of a rectangle is 3 more than twice the width. The area is 44 square inches. Find the dimensions.

4. The hypotenuse of a right triangle is 15 inches. One of the legs is 3 inches more than the other. Find the lengths of the two legs.

Remember the Pythagorian Theorem:
In a right triangle:



5. The height of a triangle is 3 cm less than the length of the base. The area is 35 square cm. Find the height and the base.