Math 90 Lecture Notes
Chapter 6

Section 6.1: Rational Expressions

A ***Rational Expression*** is any expression that can be written in the form $\frac{P}{Q}$, where P and Q are Polynomials and Q $\neq 0$. The following are examples of Rational Expression:

$\frac{2x+3}{x}$ Numerator = _______________ , Denominator = __________

$\frac{x^2-6x+9}{x^2-4}$, Numerator = ____________ , Denominator = __________

In any rational expression we must be careful to remember that the denominator can not equal zero, since division by zero is undefined!!

A. State the restrictions on the variable in the given rational expressions: (In other words, for what values of x is the expression undefined?)

1. $\frac{x-5}{x+3}$

2. $\frac{2x-7}{x^2-3x-10}$

B. A review of the properties of working with fractions/Rational Expressions:
   1. Multiplying the numerator and denominator of a Fraction/Rational Expression by the same nonzero quantity will not change the value of the rational expression.
   2. Dividing the numerator and denominator of a Fraction/Rational Expression by the same nonzero quantity will not change the value of the rational expression. (We use this property to reduce fractions).

C. **PROBLEMS** State what values of the variable make the expression undefined then reduce the expressions to lowest terms by factoring the numerator and denominator and canceling where possible.

1. $\dfrac{3x-15}{5-x}$

2. $\dfrac{x^2-9}{x^2+5x+6}$

3. $\dfrac{10a+20}{5a^2-20}$

4. $\dfrac{2x^3+2x^2-24x}{x^3+2x^2-8x}$

Section 6.2: Multiplication and Division

In this section we will multiply and divide rational expressions. Please remember that the rules for doing this is no different than the rules for multiplying and dividing fractions that contain only numbers in the numerator and denominator.

A. An example of multiplication of a fraction involving only numbers:
   1. Method one: Multiply first – then simplify!
      $\frac{3}{4}\cdot\frac{10}{21}=$

   2. Method two: Simplify first – then multiply!
      $\frac{3}{4}\cdot\frac{10}{21}=$

   3. When working with rational expressions that contain variables, it is almost always best to simplify first and then multiply later! That means factor numerator and denominator, cancel where possible, then multiply.

B. Work through the following examples to multiply rational expressions. Remember to simplify first and multiply last.
   1. $\frac{x-5}{x+2}\cdot\frac{x^2-4}{3x-15}$

   2. $\frac{3x^4}{3x-6}\cdot\frac{x-2}{x^2}$

C. Remember that to divide rational expressions, we simply multiply by the reciprocal of the fraction we are dividing by! (Sorry about the bad English)
   1. $\frac{4}{5}\div\frac{8}{9}=\frac{4}{5}\bullet\frac{9}{8}$

D. Divide the following rational expressions:

1. $\dfrac{3x-9}{x^2-x-20} \div \dfrac{x^2+2x-15}{x^2-25}$

2. $\dfrac{4x-8}{x+2} \div \dfrac{x-2}{x^2-4}$

3. $(y^2-9) \div \dfrac{y^2-2y-3}{y^2+1}$

4: $\dfrac{a^5}{b^4} \div \dfrac{a^2}{b}$

Note: Not all problems will simplify quite as much as did the examples above.

Section 6.3: Addition, Subtraction, and Least Common Denominators

In this section we will learn how to add rational expressions. Remember that a rational expression is just a "fancy" fraction. You add these just as you would numerical fractions. You must find a common denominator before you can add or subtract. Finding the common denominator will involve factoring.

A. Some numerical Examples:

1. $\frac{2}{7}+\frac{3}{7}=$

2. $\frac{5}{10}+\frac{3}{6}=$

B. Some examples involving rational expressions with common denominators:

1. $\frac{5}{x}+\frac{7}{x}=$

2. $\frac{x}{x^2-49}+\frac{7}{x^2-49}$

3. $\frac{x-5}{x^2-4x+3}+\frac{2}{x^2-4x+3}$

4. $\frac{3-2x}{x^2-6x+8}+\frac{7-3x}{x^2-6x+8}$

5. Find the least common multiple (LCM) $a+1,\ (a-1)^2,\ a^2-1$

Section 6.4: Addition and Subtraction with Unlike Denominators

In this section we will concentrate on rational expressions that involve unlike denominators. You will need to find a common denominator before adding or subtracting these fractions. Then, as before, you will work to simplify your answer.

See page 387 for a table of rules for adding and subtracting rational expressions with different denominators.

A. Some examples involving rational expressions with different denominators:

1. $\frac{2}{x}-\frac{1}{3}$

2. $\frac{7}{3x+6}+\frac{x}{x+2}$

3. $\frac{1}{x-3}+\frac{-6}{x^2-9}$

4. $\dfrac{5}{x^2-7x+12}+\dfrac{1}{x^2-9}$

5. $\dfrac{x+4}{2x+10}-\dfrac{5}{x^2-25}$

6. $1-\dfrac{1}{x}$

7. $\dfrac{y^2}{y-3}+\dfrac{9}{3-y}$

Section 6.6: Solving Rational Equations

In working with fractions that involve rational expressions, we work to rid ourselves of the fractions involved by finding common denominators. Once we do this, the equations will look very similar to ones we have already learned how to solve in previous sections of the text.

STEPS FOR SOLVING RATIONAL EQUATIONS:

1. List the restrictions.
2. Clear the denominator
3. Solve
4. Check answer with original problem

A. Some Examples:

1. $\frac{x}{3}+\frac{5}{2}=\frac{1}{2}$

2. Solve the same problem as above, but find the common denominator first:
   $\frac{x}{3}+\frac{5}{2}=\frac{1}{2}$

3. $\frac{4}{x-5}=\frac{4}{3}$

4. $1-\frac{3}{x}=\frac{-2}{x^2}$

5. $\frac{x}{x^2-9}-\frac{3}{x-3}=\frac{1}{x+3}$

6. $\frac{x}{x-3}+\frac{3}{2}=\frac{3}{x-3}$

1. Check your answer to this equation by plugging it into the original equation.
2. Surprised?????
3. What are the restrictions on the rational expressions contained in this

7. $\frac{a+4}{a^2+5a}=\frac{-2}{a^2-25}$

1. What restrictions are there on the variables for the rational expressions in this equation?
2. Be sure to check your answer(s).