Math 90 Lecture Notes
Chapter 1

Section 1.1: Introduction to Algebra

This textbook stresses Problem Solving! Solving problems is one of the main goals of mathematics. Think of mathematics as a language, and you will have a pretty good idea of what it is all about. In this section we will concentrate on translating from verbal expressions into mathematical expressions.

A. Some important ideas and terminology:

1. What is the difference between arithmetic and algebra?

2. Algebra is all about variables. What is a variable?

3. What is a constant?

4. What is a variable expression?

5. Key words and Symbols:

   i. Addition:

   ii. Subtraction:

   iii. Multiplication:

   iv. Division:

6. Equation: An equation is a number sentence with the verb "=". Equations may be true, false, or neither!

7. When studying algebra- LOOK FOR THE PATTERNS.

B. DO THESE PROBLEMS:

EVALUATION IS USED TO CHECK ANSWERS SO THIS SKILL IS VERY IMPORTANT.

1. Evaluate the following for the given value of the variable:

   a. $\frac{x-y}{6}$ for x = 23 and y = 5

   b. $\frac{5z}{y}$ for z = 9 and y = 15.

2. Translate to an algebraic expression:
   a. 7 more than Lou's weight:

   b. ***p*** subtracted from ***q***:

   c. Paula's speed minus twice the wind speed:

   d. 6 increased by x:

   e. x divided by 4:

   f. the sum of ***d*** and twice ***b:***

3. Translate each problem to an equation:
   a. Seven times what number is 2303?

   b. A carpenter charges $25 an hour. How many hours did she work if she billed a total of $53,400?

4. Determine whether the given number is a solution of the given equation:
   a. Is 75 a solution to $y + 28 = 93$

   b. Is 12 a solution to $8t = 36$

5. Substitute to find the value of each expression:
   a. A communications satellite orbiting 300 miles above the earth travels about 27,000 miles in one orbit. The time, in hours, for an orbit is given by: $t = \frac{27{,}000}{v}$, where v is the velocity, in miles per hour. How long will an orbit take at a velocity of 1,125 miles per hour?

Section 1.2: The Commutative, Associative, and Distributive Laws

In this section, we begin to learn how to manipulate algebraic expressions. Manipulating algebraic expressions will allow us to make solving equations and problem solving a lot easier.

A. Equivalent Expressions: Expressions that represent the same number. In the example below each expression equals 12:

$$4 \bullet 3,\ 4+4+4,\ 2 \bullet 6, 1+11$$

B. The Commutative Laws of Addition and Multiplication: Some examples:
Addition:
$3+4 \ \ and \ \ 4+3$
$x+y \, and \ y+x$

Multiplication:
$3 \bullet 7 \ \ and \ \ 7 \bullet 3$
$2+4y \, and \ \ 2+y \bullet 4$

USING THE MULTIPLY SYMBOL
Note that we can write 4y to indicate the product of 4 and y but if we reverse the order of the 4 and the y we should write $y \bullet 4$, showing explicit multiplication, to avoid confusion. On the other hand ab and ba both mean the product of a and b and the dot is not needed.

C. The Associative Laws of Addition and Multiplication: Some examples:
Addition: 3 + (4+ 2) and (3 + 4) + 2

Combining the Associative laws and the Commutative laws of addition:

$$1+ 8 + 3 + 9 + 2 +7 = (1+9) + (8+2) + (3+7) = 10+10+10 =30$$

D. The Distributive Law: For any numbers a, b, and c:
a(b + c) = ab + ac

1. Use the distributive property: 3(x + 5) =

2. Use the distributive property: 4(5x + 8 + 3p)

3. Use the distributive property: (x + 7)5

E. Factoring: Using the Distributive Law in Reverse: Factoring means to write as a product. An example:
Since 3(x + 5) = 3x + 15 then 3x + 15 = 3(x + 5)

F. DO THESE PROBLEMS:

1: Factor the following: (Write as a product) $2a + 2b =$

2: Factor the following: (Write as a product) $5x + 20 =$

3: Factor the following: (Write as a product) $3x + 6y + 12 =$

4: $8 + t$ is equivalent to $t + 8$ by the ____________________ law of addition.

5: $(9 + a) + b$ is equivalent to $9 + (a + b)$ by the_______________ law of addition.

6: $2(x + y)$ is equivalent to $2x + 2y$ by the ___________________ law of multiplication.

Use the commutative law of addition to arrive at an equivalent expression

7: $3a + 7b =$

Use the commutative law of multiplication to arrive at an equivalent expression

8: $x + 3y =$

9: Multiply $5(x + 2 + 3y)$

Section 1.3: Fraction Notation

This chapter is a review of addition, subtraction, multiplication, and division of fractions. This section should be a review to you. We will not spend much time here.

A. Factors and Prime Factorizations:

1. What is a factor?
2. Factor: A Factor of a number, is a number that divides into it evenly, or has no remainder. Don't forget 1 and don't forget the number itself.
   a. List all the factors of 12:

   b. List all the factors of 22:

3. Prime Numbers: A Prime Number is a number that has exactly two different factors: the number 1, and itself. NOTE: 1 IS NOT A PRIME NUMBER!
   a. List the first 10 prime numbers:

4. Natural Numbers: (Counting numbers)

5. Prime Factorization: Every composite number can be factored into the product of prime numbers, and this factorization is unique!

   a. Factor 36 into its prime factorization using a factor tree:

   b. Factor 100 into its prime factorization using a factor tree:

B. Working with Fraction - Some rules:

1. Multiplying Fractions: MULTIPLY NUMERATORS THEN MULTIPLY DENOMINATORS!

$$\frac{2}{3} \bullet \frac{5}{7} = \frac{2 \bullet 5}{3 \bullet 7} = \frac{10}{21}$$

2. Dividing Fractions: ALWAYS CONVERT A DIVISION TO THE EQUIVALENT MULTIPLICATION!

$$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \bullet \frac{7}{5} = \frac{14}{15}$$

3. Adding and Subtracting Fractions: ADDING AND SUBTRACTING FRACTIONS REQUIRE EQUAL DENOMINATORS!

$$\frac{2}{3} + \frac{5}{7} = \frac{2 \bullet 7}{3 \bullet 7} + \frac{5 \bullet 3}{7 \bullet 3} = \frac{14 + 15}{21} = \frac{29}{21}$$

4. Reducing Fractions: Canceling – See warning on p. 25. **CANCELLING REQUIRES FACTORING. IF YOU CAN'T FACTOR, YOU CAN'T CANCEL.**

$$\frac{2a + 6b}{2x + 4y} = \frac{2(a + 3b)}{2(x + 2y)} = \frac{a + 3b}{x + 2y}$$

C. DO THESE PROBLEMS:

1. Is this a set of natural numbers? 1,2,5,0,6

2. Find the prime factorization of 56.

3. Simplify $\frac{17}{51}$

4. Simplify $\frac{75}{80}$

4. Perform the indicated operation $\frac{13}{18} - \frac{4}{9}$

5. Perform the indicated operation $\frac{7}{6} \div \frac{3}{5}$

6. Perform the indicated operation $12 \div \frac{3}{7}$

Section 1.4: Positive and Negative Real Numbers

In this section we are introduced to the concept of Sets of Real Numbers and a review of negative numbers.

A. A ***Set*** is ____

B. The ***Integers*** consist of __________________________________

C. Graph the Integers below:

D. Write the ***Integers*** in Set notation: { }

D. The ***Rational Numbers*** contain all numbers that can be written as

____ .

E. Write the ***Rational Numbers*** as a set in Set Builder Notation:

____

F. The set of ***Irrational Numbers*** consists of ______________________
____

G. Examples of ***Irrational Numbers***: _____________________________

H. The Set of ***Real Numbers*** consists of __________________________

____

I. Inequalities: Define the following symbols:
   1. $<$ __________________________
   2. $>$ __________________________
   3. $\leq$ __________________________
   4. $\geq$ __________________________

J. ***Absolute Value***: The Absolute value of a number is its distance from which number? _______________

K. DO THESE PROBLEMS

Write a true sentence using either < or >.

1: 5 ________ -3   2: 0 ________ -2

3: -7 _______ -3

4: When comparing two numbers on the number line, how do you know which is the biggest?

5: Find the following Absolute Values:

a) $|-5| =$ b) $|25| =$

c) $|0| =$ d) $-|-4| =$

6: Classify the following collections of numbers by making associations between the right and left columns. That is draw a line connecting the appropriate items in the right column with the correct item in thew left column.

a. $\sqrt{2}, \sqrt{5}, \pi, \frac{\sqrt{7}}{2}$		1. PRIME
b. –3, -2 , -1, 0, 1, 2, 3		2. WHOLE
c. 4, 6, 2, 1, 0		3. INTEGER
d. 4, 5, 2, 1, 9, 23		4. REAL
e. $1.625, 3.3\overline{3}, 1.2, 2.\overline{123}$		5. RATIONAL
f. $1.4, \sqrt{3}, 2.\overline{35}, 0, -2$		6. IRRATIONAL
g. 3,5,11,13,23		7. NATURAL

7: Write the decimal notation for $-\frac{3}{8}$

8: List in order from least (on left) to greatest (on right)

$-17, -6, 5, -2, 0, 4, \frac{5}{3}, |-3|$

Section 1.5: Addition of Real Numbers

A. Two simple examples:
   1. If the temperature is 15 degrees below zero, and then drops another 10 degrees, what is the new temperature? _______________.
   2. If the temperature is 25 degrees above zero, and then drops 30 degrees, what is the new temperature? ________________.

B. Using the number line to illustrate addition of real numbers: Illustrate on the number line the addition of the following numbers: (Always start at the origin!)
   1. $-2 + (-5) =$

   2. $-7 + 5 =$

   3. $6 + (-2) =$

   4. $-11 + 7 =$

C. Rules for adding two real numbers: See page 41 of your text.

D. Combining Like Terms: Two terms with identical variable terms (including the exponent) are called 'LIKE' or 'SIMILAR'. Note the constant numbers need not be the same.

   Example: $9x - 7x = (9-7)x = 2x$

E. DO THESE PROBLEMS

1. $12+(-12)=$

2. $-3.6+1.9=$

3. $35+(-14)+(-19)+(-5)=$

4. $\frac{-3}{5}+\frac{4}{5}=$

5. $\frac{-4}{7}+\frac{-2}{7}=$

6. $-3+8x+4+\left(-10x\right)=$

7: $6x + 7x =$

8: $-9y + 11y =$

9: $4t + (-4t) =$

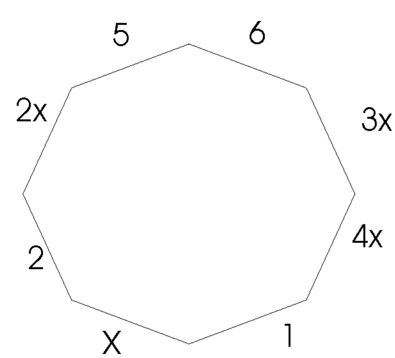
10: $8w + (-15w) =$

11: Word Problem Practice: Maya's telephone bill for July was $82. She sent a check for $50 and then made $37 worth of calls in August. What was her new balance?

12: Word Problem Practice: The new quarterback for SDSU attempted passes with the following results:

First try: 13 yard gain
Second try: 12 yard loss
Third try: 21 yard gain
Find the total gain (or loss):

9. Find the perimeter of the figure below.



Section 1.6: Subtraction of Real Numbers

Pay very close attention to this section as subtraction of real numbers can prove to be a little tricky if you are not careful.

A. Opposites and Additive Inverses: The opposite, or additive inverse, of a number "a" is written "-a". (You read this as the opposite of a).
   1. What is the opposite of 5?

   2. What is the opposite of -7?

   3. What is the opposite of 0?

   4. What is the opposite of –a?

B. The law of opposites: Any pair of opposite numbers, additive inverses, always adds up to what number? ________________

C. Let's think about a new way of thinking about subtraction. Some simple examples will help us here:
   1. You have $700 in your checking account and write a check for $200. How much do you have left?
      a. This problem used subtraction in its simplest form. (Take away)
   2. The temperature at 10:00am was 20 degrees but by 3:00pm the temperature had dropped to –10 degrees. (10 degrees below zero). What was the ***difference*** between the temperature at 10:00am and 3:00pm?
      a. This problem also used subtraction, but in a more general sense. Notice the word ***difference*** implies subtraction. This problem would translate into the following: 20 – (-10) = __________

D. A New Rule For Subtraction: (Page 48 in your text).

NOTE: ***We do not subtract in algebra. We ADD a negative number.***

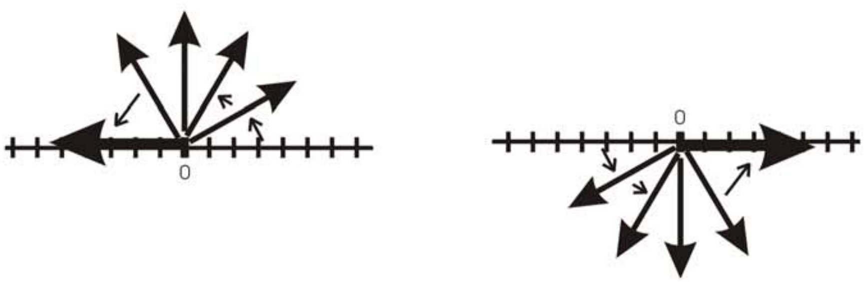
a. To subtract one number from another, simply add its opposite.
b. Always think in the following terms: $a – b = a + (-b)$
c. I know this new way of thinking of subtraction seems kind of silly, but if you can learn to think in this way, it will help you a great deal when our work becomes more intricate. Please practice this way of thinking. (Do not think of subtraction as "take – away"). I will often use this idea only when the problem is more complex and just use intuition when the problem is easier.

GETTING IT RIGHT
THE + AND - SIGNS

THE MINUS SIGN IS ATTACHED TO THE NUMBER, VARIABLE OR EXPRESSION IMMEDIATELY FOLLOWING IT.
THUS $8 - 5 = 8 + (-5)$

-5 MEANS 5 MULTIPLIED BY -1
THUS $-5 = (-1)(5)$
LET US REPRESENT 5 BY AN ARROW (VECTOR) LYING ON THE REAL NUMBER LINE, POINTING RIGHT.

THEN EACH TIME WE MULTIPLY BY -1, THE VECTOR ROTATES 180 DEG.

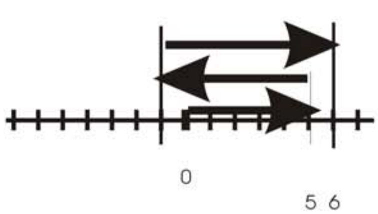


MULTIPLY 5 BY -1 ONCE RESULTS IN ONE ROTATION TO 180 DEGREES OR -5 AND THE ARROW POINTS LEFT.

MULTIPLY AGAIN BY -1 RESULTS IN ANOTHER ROTATION BACK TO 0 DEGREES OR +5 AND THE ARROW POINTS RIGHT AGAIN.

SO $(-1)(-1)(5) = +5$

THE SUMMATION OF POSITIVE AND NEGATIVE NUMBERS IS REPRESENTED BY THE SUMMATION OF POSITIVE AND NEGATIVE VECTORS. FOR INSTANCE +5 -6 +7 IS SHOWN BELOW



THE ANSWER IS +6

SO $5 -(-6)+(-5)-3 = 5+6-5-3 = 11 - 8 = 3$

E. DO THESE PROBLEMS:

1. $7 - 9 =$

2. $-7 - 8 =$

3. $0 - 10 =$

4. $-6 - (-5) =$

5. $-9 - 0 =$

6. $\frac{-1}{5} - \frac{3}{5} =$

7. $\frac{-4}{17} - \frac{9}{-17} =$

8. Simplify $14 - (-5x) + 2z - (-32) + 4z - 2x =$

9. Simplify $8x - (-2x) - 14 - (-5x) + 53 - 9x =$

10. The maximum temperature on Mars is 25°C and the minimum temperature is -125°C. What is the temperature range on Mars?

11. Subtract 37 from –21 (use a number line).

Section 1.7: Multiplication and Division of Real Numbers

A. Multiplication as repeated addition: Most students don't have a problem with multiplication when they think of it as simply repeated addition. However, as we shall see, this way of thinking of multiplication is a little harder to use when dealing with the product of two negative numbers.

1. Some simple examples:
   a. Two positive numbers: 3(7) = ____________________________

   b. One positive: 3(-7) = ____________________________________

   c. One negative: -7(3) = ___________________________________

   d. Two negatives: (-3)(-7) = ??? How do we visualize this product?

2. Earlier in the course we stated that you can find a lot of success in the study of mathematics if you can **look for patterns**. Look for a pattern in the following example:

   3(-5) =
   2(-5) =
   1(-5) =
   0(-5) =
   -1(-5) =
   -2(-5) =
   -3(-5) =

3. Another example:

   -3[5 + (-5)] = _____________________________ (Use Or. Op)

   -3[5 + (-5)] = _____________________________ Use Dist. Prop.

4. A General Rule for Multiplication: (See pages 56-57)
   When multiplying two numbers, the answer will be:

   a. Positive when: ____.

   b. Negative when: ____.

   c. Remember that these rules apply for any numbers: whole, decimal, fraction, or any combination!!!

B. Some Practice Problems with Multiplication:

1. (5)(-2) =

2. (-4)(-21) =

3. (-3)(5) =

4. (-2)(-3)(5)(2)(-2) =

C. Division of Real Numbers: We do not divide in algebra. **WE MULTIPLY BY THE RECIPROCAL.**

1. When dividing two numbers with the same sign, either both positive or both negative, the answer will be _________ .

2. When dividing two numbers with different signs, one positive and one negative, the answer will be _____________ .

3. Some sample division problems:

   a. $(26) \div (-13) =$

   b. $\frac{-200}{8} =$

   c. $\left(\frac{-2}{7}\right)\left(-\frac{5}{8}\right) =$

   d. $-\frac{5}{4} \div \left(-\frac{3}{4}\right) =$

D. Remember that division by zero is ***undefined!***

1. Noting the relationship between multiplication and division:
   a. $\frac{10}{5}=2$ because 10 = 2(5) Both of these equations are true.

   b. $\frac{0}{5}=0$ because 0 = 0(5)

   c. Notice the problem we encounter when we try to divide by zero: (Here, we will let the letter "n" stand for any number that "could" be a solution to this problem)
   Suppose $\frac{5}{0}=n$
   If this is true, then the following should also be true: 5 = n(0) !

   But we already know that any number times zero is zero, it can never be five (5). So this is another way to convince yourself that division by zero is undefined and therefore not allowed.

   d. What is the solution to the following problem?

$$\frac{(5)(-3)}{2+(-7)+5}=$$

   e. What single word best describes the reciprocal of zero

Section 1.8: Exponential Notation and Order of Operations

In this section we will learn about the use of exponents, and review the topic of "Order of Operation".

Exponential notation:

1. Expand the following: $5^3 =$

2. In the expression above, the "3" is called an __________________ , and the "5" is called the ____________ .

3. In this expression the exponent tells us to multiply the base by itself _____times.

Order of Operation: Look at the guidelines on page 65 of your text!

Parenthesis, Exponents, (Multiply & Divide), (Add and Subtract)

A few important notes about the use of "-1" and subtraction:

1. When you multiply a number by negative 1, the result is the opposite of that number.

   For example: -1(3) = ____________

   Another example: (-1)(-5) = ____________

2. Finding the opposite of a sum:

   Evaluate the following: -(x + y) =

   Evaluate the following: -(2 + 3) =

   Evaluate the following: -(7 – 3) =

   Simplify the following: -(x – y) =

For the rest of these notes, we are going to practice all the skills we have learned in this chapter so far. We will be simplifying Algebraic Expressions by correctly using the order of operation, methods of adding and subtracting positive and negative numbers, combining like terms, and correctly interpreting the use of the negative sign. Practice this section very carefully!

DO THESE PROBLEMS:

1. $(-7)^2 =$

2. $-7^2 =$

3. $(5t)^2 =$

4. $14-2\cdot 6+7=$

5. $3(-10)^2-8\div 2^2 =$

6. $9-(3-5)^3-4=$

7. $\dfrac{5^2-3^2}{2\cdot 6-4}$

8. $6a\div 12a^3$, Evaluate for a = 2

Rewrite the following without using parentheses:

9. $-(3x+5)=$

10. $-(8x^3-6x+5)=$

12. Simplify: $-8a^2+5ab-12b^2-6(2a^2-4ab-10b^2)$