Trigonometry, Mckeague, 6th edition
Homework Proof

5.3 #53

Prove: $\dfrac{1-\tan x}{1+\tan x}=\dfrac{1-\sin(2x)}{\cos(2x)}$.

Start with:

$$\frac{1-\tan x}{1+\tan x}$$

$$=\frac{1-\left(\dfrac{\sin x}{\cos x}\right)}{1+\left(\dfrac{\sin x}{\cos x}\right)}=\frac{\dfrac{\cos x}{\cos x}-\left(\dfrac{\sin x}{\cos x}\right)}{\dfrac{\cos x}{\cos x}+\left(\dfrac{\sin x}{\cos x}\right)}=\frac{\dfrac{\cos x-\sin x}{\cos x}}{\dfrac{\cos x+\sin x}{\cos x}}$$

$$=\frac{\cos x-\sin x}{\cos x}\cdot\frac{\cos x}{\cos x+\sin x}$$

$$=\frac{\cos x-\sin x}{\cos x+\sin x}$$

Now this is a problem. I don't have any identities to use here.

I need squares to get some possiblities for identities.

One way to make squres is multiple by the conjugate.

$$=\frac{\cos x-\sin x}{\cos x+\sin x}\cdot\left(\frac{\cos x-\sin x}{\cos x-\sin x}\right)$$

The "fancy one" keeps the expression equivalent still.

$$=\frac{\cos^2 x-2\sin x\cos x+\sin^2 x}{\cos^2 x-\sin^2 x}$$

The result of FOILing. Note all the squares and identities.

$$=\frac{1-2\sin x\cos x}{\cos(2x)}.$$

Therefore, $\dfrac{1-\tan x}{1+\tan x}=\dfrac{1-\sin(2x)}{\cos(2x)}$.